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An equivalent classical plate model of corrugated structures

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ABSTRACT

An equivalent classical plate model of corrugated structures is derived using the variational asymptotic method. Starting from a thin shell theory, we carry out an asymptotic analysis of the strain energy in terms of the smallness of a single corrugation with respect to the characteristic length of macroscopic deformation of the corrugated structure. We obtained the complete set of analytical formulas for effective plate stiffnesses valid for both shallow and deep corrugations. These formulas can reproduce the well-known classical plate stiffnesses when the corrugated structure is degenerated to a flat plate. The extension–bending coupling stiffnesses are obtained the first time. The complete set of relations are also derived for recovering the local fields of corrugated structures.

1. Introduction

Corrugated structures have been widely used in civil, automotive, naval and aerospace engineering, to name only some, diaphragms for sensing elements, fiberboards, folded roofs, container walls, sandwich plate cores, bridge decks, ship panels, etc. (Andreeva, 1966; Mccready and Katz, 1939; Seaquist, 1964; Baum et al., 1981; Carlsson et al., 2001; Liang et al., 2001; Davalos et al., 2001; Buannic et al., 2003; Aboura et al., 2004; Talbi et al., 2009; Haj-Ali et al., 2009; Viguié et al., 2011). Recently, corrugated structures are also applied for flexible wings or morphing wings (Yokozeki et al., 2006; Gentilinia et al., 2009; Thill et al., 2010) due to their unique characteristics of having orders of magnitude different stiffnesses in different directions.

Although commercial codes allow one to analyze corrugated structures by meshing all the corrugations using shell elements or solid elements, it is not a practical way to finish prototype in a timely manner as it requires significant computing time, particularly if the structure is formed by hundreds or thousands of corrugations. The common practice in analysis of corrugated structures is to model it as an equivalent flat plate, which is possible if the period of corrugation is much smaller than the characteristic length of macroscopic deformation of the structure (see Fig. 1). For example, to model the corrugated structure using the Kirchhoff plate model, also called the classical plate model, we need to obtain the following strain energy by analyzing a single corrugation:

$$J = \frac{1}{2}\begin{Bmatrix} \epsilon_{xx} \\ \epsilon_{yy} \\ 2\epsilon_{xy} \\ \kappa_{xx} \\ \kappa_{yy} \\ 2\kappa_{xy} \end{Bmatrix}^T \begin{Bmatrix} A_{11} & A_{12} & 0 & B_{11} & B_{12} & 0 \\ A_{12} & A_{22} & 0 & B_{12} & B_{22} & 0 \\ 0 & 0 & A_{66} & 0 & 0 & B_{66} \\ B_{11} & B_{12} & 0 & D_{11} & D_{12} & 0 \\ B_{12} & B_{22} & 0 & D_{12} & D_{22} & 0 \\ 0 & 0 & B_{66} & 0 & 0 & D_{66} \end{Bmatrix} \begin{Bmatrix} \epsilon_{xx} \\ \epsilon_{yy} \\ 2\epsilon_{xy} \\ \kappa_{xx} \\ \kappa_{yy} \\ 2\kappa_{xy} \end{Bmatrix} \quad (1)$$

where x, y are the two in-plane coordinates describing the equivalent plate, $\epsilon_{xx}, \epsilon_{yy}, \epsilon_{xy}$ the membrane strains, $\kappa_{xx}, \kappa_{yy}, \kappa_{xy}$ the curvature strains, A_{ij}, D_{ij} and B_{ij} represent extension stiffnesses, bending stiffnesses, and extension–bending couplings, respectively. The stiffness matrix in Eq. (1) could be in general populated for an equivalent plate model of general corrugated structures. However, it will be shown later that some of the stiffness constants vanish as shown in Eq. (1) for a corrugated structure made of a single isotropic material.

The literature is rich in equivalent plate modeling of corrugated structures with the first treatment known to the authors dated 1923 (Huber, 1923) and a very recent treatment appeared in 2013 (Bartolozzi et al., 2013). Various methods with different levels of sophistication were used in numerous studies. Generally speaking, existing methods can be generally classified either as engineering approaches based on various assumptions or asymptotic approaches based on asymptotic analysis of governing differential equations of a shell theory. Most methods fall in the category of engineering approaches which invoke various assumptions for boundary conditions and force/moment distribution within the corrugated structure. For a given state of constant strain, the actual

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(or assumed) distributions of forces and moments within the corrugated structure will be determined. Then force or energy equivalence is used to derive the corresponding stiffness constants (see Briassoulis, 1986; Xia et al., 2012; Bartolozzi et al., 2013 and references cited therein). Although both analytical approach and finite element analysis can be used to predict these stiffness constants, the analytical approach has the advantage of providing a set of close-form expressions in terms of the material and geometry characteristics of the corrugated structure while the finite element analysis predicts values which are valid for a specific corrugated structure. Asymptotic approaches exploit the smallness of a single corrugation with respect to characteristic length of macroscopic deformation of the corrugated structure (Andrianov et al., 1998; Manevich et al., 2002; Arkhangelskii and Gorbachev, 2007; Andrianov et al., 2009). Substituting asymptotic expansion of the field variables into the governing differential equation of the shell theory, a series of system of governing differential equations corresponding to different orders can be solved to find the relationship between the equivalent plate and the corrugated structure. Because different methods are used to treat this problem, it is not surprising that different results are obtained in previous studies, which will summarized and compared here.

2. Results

To facilitate the comparison of different results in the literature, we need to set up the necessary notations. Let x be the Cartesian coordinate in the corrugation direction and ε the projected length of the corrugation Fig. 2. We denote by $X = \frac{x}{\varepsilon}$, the dimensionless "cell coordinate". Within a cell, X changes between $-1/2$ and $1/2$. For any parameter, f, changing within a cell, $\langle f \rangle \equiv \int_{-\frac{1}{2}}^{\frac{1}{2}} f(X)dX$. The shape of the corrugation is described by the $x_3(X)$ which is a periodic function with the period unity. Without loss of generality, one can set

$$\langle x_3 \rangle = 0, \tag{2}$$

by shifting the observer's frame in the vertical direction. Let us also denote

$$x_3 = \varepsilon\phi(X), \quad \varphi = \frac{dx_3(x)}{dx} = \frac{d\phi(X)}{dX}, \quad a = 1 + \varphi^2, \tag{3}$$

we can compute the arc-length of the corrugation S and the moment of inertia along the corrugation direction I_y as

$$S = \varepsilon\langle\sqrt{a}\rangle, \quad I_y = h\varepsilon^2\langle\phi^2\sqrt{a}\rangle. \tag{4}$$

2.1. Results from previous studies

Seydel (1931) followed Huber (1923) and obtained the following formulas for the equivalent bending stiffnesses

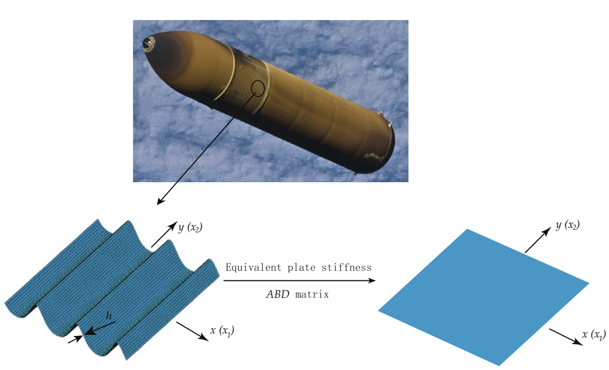

Fig. 1. Equivalent plate modeling of corrugated structures.

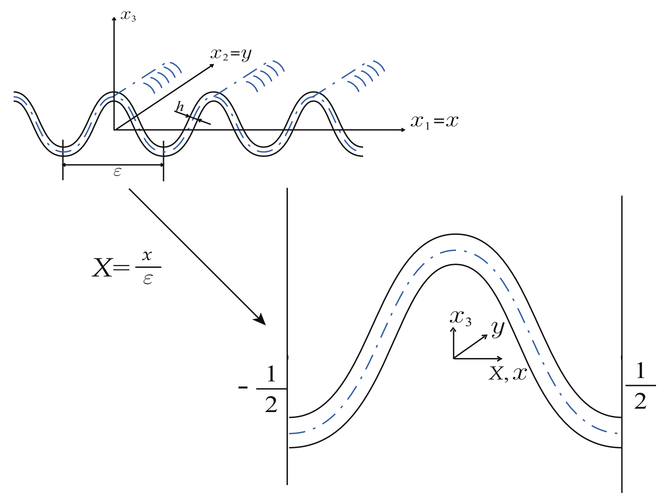

Fig. 2. Shell geometry and unit cell.

$$D_{11} = \frac{\varepsilon}{S}\frac{Eh^3}{12(1-\nu^2)}, \quad D_{12} = 0, \quad D_{22} = EI_y, \quad D_{66} = \frac{S}{\varepsilon}\frac{Eh^3}{24(1+\nu)}. \tag{5}$$

Here h denotes the thickness Fig. 3. It is assumed that the corrugated plate is made of isotropic elastic material with the Young's modulus E, and the Poisson's ratio ν. These results are also widely cited in textbooks (Szilard, 1974; Bending et al., 1976; McFarland et al., 1972). In later works, approximations for S and I_y for different corrugated shapes were obtained (Lekhnitskii, 1968; Szilard, 1974; Lau, 1981; Lee, 1981). A review of different approximate formulas of S and I_y for various corrugation shapes can be found in Luo et al. (1992). This is not needed as it is easy to evaluate the two integrals in Eq. (4) accurately for any given corrugated shape using computers nowadays.

Later, Briassoulis (1986) proposed the following modified relations

$$D_{11} = \frac{\varepsilon}{S}\frac{Eh^3}{12(1-\nu^2)}, \quad D_{12} = \nu D_{11},$$
$$D_{22} = \frac{EhT^2}{2} + \frac{Eh^3}{12(1-\nu^2)}, \quad D_{66} = \frac{Eh^3}{24(1+\nu)}. \tag{6}$$

Here T is the rise of the corrugations measured to middle surface as shown in Fig. 3. Briassoulis correctly recognized D_{12} due to the Poisson's effect. However, as will be shown later, the formulas for D_{22} and D_{66} are not correct. The expression for D_{22} is obtained by assuming a sinusoidal corrugated profile, $x_3 = T\sin(2\pi x/\varepsilon)$. Briassoulis's relations are also used in Liew et al. (2006, 2009) and

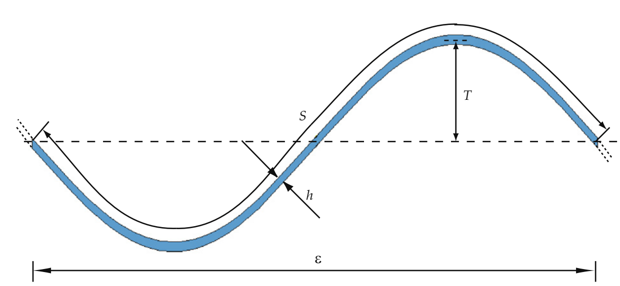

Fig. 3. Unit cell of a corrugated structure (sinusoidal shape is used for illustration).