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Fractured water injection wells: Pressure transient analysis

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ABSTRACT

In this paper we study the pressure drop in a hydraulic fracture after shut-in of a water injection well. The pressure transient behavior depends on fracture closure, lateral stress, rock elasticity and fracture fluid leak-off. Under the assumption that horizontal cross-sections of a vertical fracture do not depend on the vertical variable, we formulate a mathematical model which allows for determination of both pore pressure and elastic rock displacements jointly with the fracture aperture and fracture fluid pressure. An analytical consideration is performed for the case of an ideal very long fracture with the same aperture along its full length. In the general case, fracture closure is analyzed numerically.

1. Introduction

It is well established that well fracturing may occur while a large volume of water is injected to maintain an oil production pressure. One way to determine the dimensions of the induced fractures is to analyze the pressure transient data for these wells (Cinco-Ley and Samaniego, 1981). A number of papers is dedicated to the injection fall-off test analysis which offers one of the cheapest ways to determine the dimensions of induced fractures. The goal of the present paper is to contribute to this study.

The theories developed (Nolte, 1986) are not sufficiently advanced to put together fracture closure, pressure distribution along the fracture, leak-off rate through the fracture faces, regional stresses, etc. It is due to the lack of a good mathematical model that one should formulate a hypothesis that the flow near the crack is split into "storage", "linear", "bilinear" and "radial" regimes in the course of time, without knowledge of the regime durations (Economides and Nolte, 2000). As for the Khristianovich–Zheltov–Geertsma–de Klerk (KGD) model and the Perkins–Kem–Nordgren (PKN) model (Adachi et al., 2007) they permit to relate the fracture aperture with the fracture pressure but under the strong assumption that rock stress field does not depend on pore pressure distribution. We do not make assumptions on flow regimes; in our approach, the flow regime and the solid matrix deformations interact and can be defined only simultaneously.

Here, we study a flow of a fluid between the fracture faces jointly with the flow through a porous medium taking into account that the medium is elastic. In this way we find directly the pore pressure, the rock stress and the fracture pressure without any simplified leak-off hypotheses like the Carter formula (Economides and Nolte, 2000). We restrict ourselves to the case of a fracture of fixed size. We do not concern fracture stimulation; our goal is rather to relate the fracture closure with the pressure drop after injection shut-in.

2. A mathematical model

We consider a vertical hydraulic fracture of fixed height $2H$ and fixed length $2L$ extending along the x- axis with z being the vertical variable, Fig. 1. The fracture is open in the y-direction due to the fluid injection at the center of the coordinate system (x, y). In what follows, we restrict ourselves to the displacements in the plane $z = 0$, Fig. 2, assuming that all the cross-sections by the planes $z = H_1$, $|H_1| \leqslant H$, are effectively identical.

The poroelastic material near the fracture is considered to be a homogeneous permeable medium which is governed by Biot (1956) equations. At the instant t, each infinitesimal volume centered at the point $\mathbf{x}$ is characterized by the solid phase displacement $\mathbf{u}(t, \mathbf{x})$, the fluid phase displacement $\mathbf{v}(t, \mathbf{x})$ and the pore pressure $p(t, \mathbf{x})$.

It is assumed that pores are saturated by a single-phase Newtonian fluid with efficient viscosity and efficient density which are chosen to be representative of the multi-phase real fluid. Many authors apply the hypothesis that the injected fluid and the formation fluid are effectively the same (Adachi et al., 2007). We also apply such an assumption.

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Nomenclature

H	half of fracture height, cm	ϕ	formation porosity, dimensionless
L	half of fracture length, cm	ϕ_c	fracture porosity, dimensionless
h	fracture depth, cm	ρ_f	pore fluid density, g/cm^3
p	pore pressure, Pa	ρ_s	solid matrix density, g/cm^3
$\mathbf{u}$	solid phase displacement vector of poroelastic medium, cm	k_r	formation permeability, mD
$\mathbf{v}$	fluid phase displacement vector of poroelastic medium, cm	k_c	fracture permeability, mD
$\mathbf{q}$	Darcy velocity vector, cm/s	η_r	formation fluid viscosity, cp
τ	effective stress tensor of poroelastic medium, Pa	η_c	fracture fluid viscosity, cp
$\mathcal{E}$	effective strain tensor of poroelastic medium, dimensionless	E	effective Young modulus of poroelastic medium, Pa
u	displacement of poroelastic medium along the x-variable, cm	ν	effective Poisson ratio of poroelastic medium, dimensionless
v	displacement of poroelastic medium along the y-variable, cm	μ	effective shear modulus of poroelastic medium, Pa
w	fracture aperture, cm	λ	effective bulk modulus of poroelastic medium, Pa
Ω	vicinity domain of fracture	α	Biot coefficient, dimensionless
R	domain radius, cm	S_ϵ	fluid yielding capacity coefficient, Pa^{-1}
Γ_l	lateral boundary of domain	λ_l	lateral stress coefficient, dimensionless
Γ_c	fracture surface	σ_∞	lateral load, Pa
		p_∞	lateral fluid pressure, Pa
		p_g	medium weight, Pa
		Q_v	total injection rate, m^3/day

We introduce the Darcy velocity $\mathbf{q} = \mathbf{w}_t$, where $\mathbf{w} = \phi(\mathbf{v} - \mathbf{u})$ and ϕ is the porosity. It is shown by Shelukhin and Eltsov (2012,) that slow flows are governed by the quasi-static Biot equations:

$$\operatorname{div}\tau = \rho\mathbf{g}, \quad \mathbf{q} = -\frac{k_r}{\eta_r}\nabla p, \quad \rho = \phi\rho_f + (1-\phi)\rho_s,$$
$$(\operatorname{div}\tau)_i \equiv \partial\tau_{ij}/\partial x_j,$$

where τ is the effective stress tensor, k_r is the permeability, and η_r is the pore fluid viscosity, $\mathbf{g}$ is the gravitation acceleration, ρ_f and ρ_s are the fluid phase density and the solid phase density respectively. In the Biot theory, the tensor τ is defined as follows

$$\tau = \lambda\epsilon \cdot I + 2\mu\mathcal{E}(u) - \alpha p \cdot I, \quad \epsilon = \operatorname{tr}\mathcal{E}(u) \equiv \mathcal{E}(u)_{ii} \equiv \operatorname{div}\mathbf{u}, \tag{1}$$

where $I_{ij} = \delta^i_j$, $\mathcal{E}(u)$ is the strain tensor related to the field $\mathbf{u}$, $2\mathcal{E}(u)_{ij} = \partial u_i/\partial x_j + \partial u_j/\partial x_i$, α is the Biot coefficient, λ and μ are the elasticity moduli, $x = x_1,\ y = x_2,\ z = x_3$.

Generally, the porosity ϕ is a function of ϵ and p, this is why one can write the equality (Biot, 1955)

$$\frac{\partial\phi}{\partial t} = \alpha\frac{\partial\epsilon}{\partial t} + S_\varepsilon\frac{\partial p}{\partial t},$$

where S_ε is the fluid yielding capacity coefficient. Due to incompressibility of the pore fluid, the fluid mass conservation law becomes

$$\frac{\partial\phi}{\partial t} + \operatorname{div}\mathbf{q} = 0.$$

By excluding $\mathbf{q}$, one obtains that, outside the fracture, the flow is defined by $\mathbf{u}$ and p which satisfy the system

$$\operatorname{div}\tau = \rho\mathbf{g}, \quad S_\varepsilon\frac{\partial p}{\partial t} = \operatorname{div}\left(\frac{k_r}{\eta_r}\nabla p - \alpha\frac{\partial\mathbf{u}}{\partial t}\right). \tag{2}$$

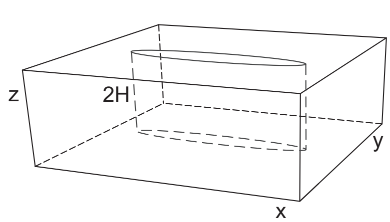

Fig. 1. Fracture geometry.

In application, $S_\epsilon = K_b B_0 \alpha^{-1}$, where K_b is the bulk modulus of rock frame drained of any pore-filling fluid, B_0 is the Skempton coefficient; the modulus λ can be calculated by the formula (Gassmann, 1951) $\lambda = P + Q - 2\mu$, where

$$P = \frac{4\mu}{3} + \frac{(1-\phi)[(1-\phi)K_s - K_b] + \phi K_s K_b/K_f}{(1-\phi) - K_b/K_s + \phi K_s/K_f},$$

$$Q = \frac{\phi[(1-\phi)K_s - K_b]}{(1-\phi) - K_b/K_s + \phi K_s/K_f}.$$

Here, K_f and K_s are the bulk moduli of the pore fluid and mineral matrix respectively, and μ is the shear modulus.

Observe that the moduli λ and μ can be obtained by other means. Given a Young modulus E and a Poisson ratio ν for the fluid-saturated rock, one can use the formulas

$$\lambda = \frac{\nu E}{(1+\nu)(1-2\nu)}, \quad \mu = \frac{E}{2(1+\nu)}.$$

For simplicity, we assume that the displacements and pressure are symmetrical relative to the plane $y = 0$. Since we study displacements in the plane $z = 0$ only, we assume the displacement vector $\mathbf{u}$ to be two-dimensional, $\mathbf{u} = (u_1, u_2) \equiv (u, v)$. As is customary in the theory of linear elasticity, we assume that the fracture lies in the line $y = 0$ and occupies the segment $-L < x < L$, with $w(t,x) = v|_{y=0}$ being half the fracture aperture. Introducing a $2 \times 2-$ matrix $\mathcal{E}(u)_{ij} = 0.5(\partial u_i/\partial x_j + \partial u_j/\partial x_i)$, $(i,j \neq 3)$, and defining a $2 \times 2-$ matrix τ_{ij} by formula (1), we project Eq. (2) onto the plane $z = 0$ to find that the two-dimensional displacement $\mathbf{u}$ and the pressure p satisfy the equations

$$\operatorname{div}\tau = 0, \quad S_\varepsilon\frac{\partial p}{\partial t} = \operatorname{div}\left(\frac{k_r}{\eta_r}\nabla p - \alpha\frac{\partial\mathbf{u}}{\partial t}\right), \quad (x,y) \in \Omega, \tag{3}$$

where $L < a$ and

$$\Omega = \{(x,y) : |x| < a, 0 < y < b\}.$$

At $\Gamma_l = \partial\Omega \cap \{y > 0\}$, a load $\sigma_\infty = \lambda_l p_g$ is applied and a pore pressure p_∞ is prescribed:

$$\Gamma_l: \quad p = p_\infty, \quad \mathbf{n}\cdot\tau\langle\mathbf{n}\rangle = -\sigma_\infty, \quad \mathbf{s}\cdot\tau\langle\mathbf{n}\rangle = 0,$$
$$(\tau\langle\mathbf{n}\rangle)_i \equiv \tau_{ij}n_j. \tag{4}$$