International Journal of Solids and Structures 51 (2014) 2167–2182

Contents lists available at ScienceDirect

International Journal of Solids and Structures

journal homepage: www.elsevier.com/locate/ijsolstr

Transient thermal shock fracture analysis of functionally graded piezoelectric materials by the extended finite element method

Peng Liu [a], Tiantang Yu [a,*], Tinh Quoc Bui [b,*], Chuanzeng Zhang [b], Yepeng Xu [a], Chee Wah Lim [c]

[a] Department of Engineering Mechanics, Hohai University, Nanjing 210098, PR China
[b] Department of Civil Engineering, University of Siegen, Paul-Bonatz-Str. 9-11, D-57076 Siegen, Germany
[c] Department of Civil and Architectural Engineering, City University of Hong Kong, Tat Chee Avenue, Kowloon, Hong Kong

ARTICLE INFO

Article history:
Received 2 August 2013
Received in revised form 13 February 2014
Available online 7 March 2014

Keywords:
Dynamic fracture
X-FEM
Functionally graded piezoelectric materials
Thermal shock
Dynamic thermal stress intensity factors
Dynamic electrical displacement intensity factor

ABSTRACT

Transient thermal dynamic analysis of stationary cracks in functionally graded piezoelectric materials (FGPMs) based on the extended finite element method (X-FEM) is presented. Both heating and cooling shocks are considered. The material properties are supposed to vary exponentially along specific direction while the crack-faces are assumed to be adiabatic and electrically impermeable. A dynamic X-FEM model is developed in which both Crank–Nicolson and Newmark time integration methods are used for calculating transient responses of thermal and electromechanical fields respectively. The generalized dynamic intensity factors for the thermal stresses and electrical displacements are extracted by using the interaction integral. The accuracy of the developed approach is verified numerically by comparing the calculated results with reference solutions. Numerical examples with mixed-mode crack problems are analyzed. The effects of the crack-length, poling direction, material gradation, etc. on the dynamic intensity factors are investigated. It shows that the transient dynamic crack behaviors under the cooling shock differ from those under the heating shock. The influence of the thermal shock loading on the dynamic intensity factors is significant.

1. Introduction

The effective conversion of the electrical energy into mechanical energy and vice versa has led the smart piezoelectric materials to important applications in many engineering disciplines such as smart devices, micro-electromechanical systems (MEMS) and intelligent technologies. In order to improve the reliability of piezoelectric materials, functionally graded piezoelectric materials (FGPMs) have been developed by introducing the concept of gradient distribution to the piezoelectric materials (Wu et al., 1996). Consequently, some or all properties such as elastic, piezoelectric, dielectric, etc. may vary along one specific direction based on a particular gradation law. The key advantages of the FGPMs are the reduction of mechanical stress concentration, increased bonding strength, improved stress redistribution, and so on. However, the FGPMs devices and structures are usually operated in frequently changing thermal environments, thus it is important to understand the dynamic behaviors of cracked FGPMs exposed to a thermal shock.

In the early 1980s, the transient dynamic fracture behaviors of an edge-cracked plate under thermal shock were investigated (Nied, 1983). With the emergence of functionally graded materials (FGMs), many authors later studied the thermal dynamic crack analyses in FGMs by using different numerical methods, including the finite element method (FEM) (Zamani and Eslami, 2009), the boundary element method (BEM) (Hosseini-Tehrani et al., 2001; Ekhlakov et al., 2012), and the meshless methods (Sladek et al., 2008). The multiple cracking approach was developed for analyzing the thermal shock resistance and intensity release of FGMs and ferroelectric materials (Han and Wang, 2005; Wang and Li, 2005; Wang et al., 2004). The effect of surface pre-crack morphology on the fracture of thermal barrier coatings under thermal shock (Zhou and Kokini, 2004), and the influence of radial stress on the poling behavior of lead zirconate titanate ceramics (Njiwa et al., 2007) were also investigated. Recently, some efforts have been made to investigate the thermally induced fracture in piezoelectricity including the static thermal fracture problems (Niraula and Noda, 2002; Ueda, 2003, 2006a,b; Wang and Noda, 2004) and the transient thermal fracture problems (Wang and Mai, 2002; Ueda, 2006c; Sladek et al., 2010a).

Some other works investigated the thermally induced static fracture in FGPMs using the integral transform methods (Wang

* Corresponding authors. Tel.: +49 2717402836; fax: +49 2717404074 (T.Q. Bui).
E-mail addresses: tiantangyu@hhu.edu.cn (T. Yu), tinh.buiquoc@gmail.com (T.Q. Bui).

http://dx.doi.org/10.1016/j.ijsolstr.2014.02.024
0020-7683/

and Noda, 2001; Ueda, 2003, 2007, 2008, 2012; Ueda and Ashida, 2009), in which an infinite or a semi-infinite functionally graded piezoelectric medium is often considered. However, results on the transient thermal fracture behaviors of the FGPMs are very rare in the literature. To our knowledge, the only related paper is on the dynamic thermal fracture behaviors of continuously nonhomogeneous magneto-electro-thermo-elastic solids by the meshless local Petrov–Galerkin (MLPG) method (Sladek et al., 2010b). To better understand the failure patterns of FGPMs under thermal environments, it is important to develop efficient and accurate numerical methods for the evaluation of fracture parameters in FGPMs under thermal shock loading. To serve this purpose, domain form of the interaction integrals for the computation of the stress intensity factors and electric displacement intensity factor in cracked FGPMs under thermoelectromechanical loading was derived (Rao and Kuna, 2010).

The extended finite element method (X-FEM) is a quite new numerical method which can effectively overcome the drawbacks of the conventional FEM in modeling the discontinuities (Belytschko and Black, 1999; Moës et al., 1999). The principle of the X-FEM is to enrich the standard finite element approximation by some functions around the discontinuity in the framework of partition of unity (PU). Thus, the geometry of discontinuity is independent of the finite element computation mesh. In the past decades, there are a great number of studies concerning the improvement or applications of the X-FEM for various discontinuous problems (Belytschko et al., 2003; Samaniego and Belytschko, 2005; Gerstenberger and Wall, 2008; Zhang et al., 2012; Groß and Reusken, 2007; Sukumar et al., 2001; Yu, 2011). Recently, the X-FEM models for analyzing crack in piezoelectric materials (Béchet et al., 2009; Bhargava and Sharma, 2011, 2012; Bui and Zhang, 2012; Sharma et al., 2013) were developed. In these researches, two types of crack-tip enrichment functions were used, i.e., the standard fourfold crack-tip enrichment functions used for isotropic elasticity and the sixfold crack-tip enrichment functions derived from the asymptotic expansion around a crack-tip in piezoelectric materials; and the results show that enrichment functions used in isotropic materials are also efficient to the fracture of piezoelectric materials. Some major advantages in the application of the standard fourfold enrichment functions to the piezoelectric materials are: the implementation is significantly simpler; the computational cost is cheaper; the accuracy between two sets is similar.

In our previous works, the authors have developed an X-FEM model associated with the standard fourfold crack-tip enrichment functions, and investigated the transient behaviors of the dynamic intensity factors (DIFs) in FGPMs under mechanical and electrical impact loading (Liu et al., 2013). To the best knowledge of the authors, studies concerning the transient thermal shock facture analysis in FGPMs using the X-FEM are not reported in the literature. The main objective of this research work is to further extend our previous X-FEM model to numerically study the transient dynamic fracture behaviors of cracked FGPMs subjected to thermal shock. In this study, the material properties are supposed to be varying exponentially along a given direction, the crack-faces are assumed to be adiabatic and electrically impermeable, and the standard fourfold crack-tip enrichment functions are used in the X-FEM. Furthermore, the Crank–Nicolson and Newmark time integration schemes are used to achieve the dynamic responses of the thermal and electromechanical fields respectively, and the dynamic intensity factors (DIFs) including the dynamic thermal stress intensity factors (DTSIFs) and the dynamic electric displacement intensity factor (DEDIF) are evaluated by using the contour interaction integral technique. The effects of the crack-length, the material gradation direction, the material gradient index, and the poling direction, etc. on the DIFs are investigated in details through numerical examples including mode-I and mixed-modes.

After the introduction, Section 2 briefly presents some basic equations for cracked FGPMs subjected to a thermal shock. The X-FEM for modeling crack problems in FGPMs under thermal shock is then described in Section 3. The DTSIFs and DEDIF are derived by using the interaction integral in Section 4. Numerical validation is presented in the subsequent section, while the numerical results derived from the proposed X-FEM are presented in Section 6. Some conclusions drawn from the present study are given in Section 7.

2. Basic equations for functionally graded piezoelectric materials

The governing equations and the boundary conditions of FGPMs under small strain assumptions and thermal shock loading are briefly given in this section.

- Constitutive equations

$$\begin{aligned} \sigma_{ij} &= C_{ijks}\varepsilon_{ks} - e_{sij}E_s - \gamma_{ij}\Delta T \\ D_i &= e_{iks}\varepsilon_{ks} + h_{is}E_s + p_i\Delta T \end{aligned} \tag{1}$$

in which $\sigma_{ij}, \varepsilon_{ks}, C_{ijks}$ and e_{sij} are the Cauchy stress tensor, strain tensor, Hooke's elasticity tensor, and piezoelectric tensor, respectively; $E_s, \gamma_{ij}, \Delta T$ are the electric field, stress–temperature modulus, and temperature change; D_i, h_{is} and p_i represent the electrical displacements, dielectric tensor and pyroelectric coefficients.

The stress–temperature modulus γ_{ij} can be expressed as

$$\gamma_{ij} = C_{ijkl}\eta_{kl} \tag{2}$$

where η_{kl} are the linear thermal expansion coefficients.

The non-homogeneous material properties are supposed to vary continuously either in x- or y-coordinate, and an exponential law for the elastic, piezoelectric, dielectric tensors, stress–temperature modulus, pyroelectric coefficients and linear thermal expansion coefficients are employed. In the case of the material gradation in the x-direction, they are described by

$$C_{ijks}(\boldsymbol{x}) = C^0_{ijks}e^{\beta x} \tag{3}$$

$$e_{sij}(\boldsymbol{x}) = e^0_{sij}e^{\beta x} \tag{4}$$

$$h_{is}(\boldsymbol{x}) = h^0_{is}e^{\beta x} \tag{5}$$

$$\gamma_{is}(\boldsymbol{x}) = \gamma^0_{is}e^{\beta x} \tag{6}$$

$$p_i(\boldsymbol{x}) = p^0_i e^{\beta x} \tag{7}$$

$$\eta_{kl}(\boldsymbol{x}) = \eta^0_{kl}e^{\beta x} \tag{8}$$

where $C^0_{ijks}, e^0_{sij}, h^0_{is}, \gamma^0_{is}, p^0_i$, and η^0_{kl} are material properties at $x = 0$, and β is the gradient index.

In the plane-strain problems, the generalized constitutive equations may be expressed in matrix form as (Sladek et al., 2010b)

$$\begin{Bmatrix} \sigma_{11} \\ \sigma_{33} \\ \sigma_{13} \\ D_1 \\ D_3 \end{Bmatrix} = \begin{bmatrix} C_{11} & C_{13} & 0 & 0 & -e_{31} \\ C_{13} & C_{33} & 0 & 0 & -e_{33} \\ 0 & 0 & C_{44} & -e_{15} & 0 \\ 0 & 0 & e_{15} & h_{11} & 0 \\ e_{31} & e_{33} & 0 & 0 & h_{33} \end{bmatrix} \begin{Bmatrix} \varepsilon_{11} \\ \varepsilon_{33} \\ 2\varepsilon_{13} \\ E_1 \\ E_3 \end{Bmatrix} + \begin{bmatrix} -\gamma_{11} \\ -\gamma_{33} \\ 0 \\ p_1 \\ p_3 \end{bmatrix} \Delta T \tag{9}$$