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Mechanical experiments to identify homogeneous bodies

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ABSTRACT

All bodies are inhomogeneous at some scale but experience has shown that some of these bodies can be idealized as a homogeneous body. Here we examine which bodies can be idealized as a homogeneous body when they are subjected to a non-dissipative mechanical process. This is done by studying circumstances in which an inhomogeneous body admits pure stretch homogeneous deformations. Then, we devise experiments wherein these circumstances are prevented. If homogeneous deformation is observed in these devised experiments, the body could be modeled as a homogeneous body. We limit our analysis to a class of isotropic elastic bodies deforming from a stress free reference configuration whose Cauchy stress is explicitly related to left Cauchy–Green deformation tensor. It is further assumed that the constitutive relation is differentiable function of the position vector of material particles in the stress free reference configuration. Then, we find that a cuboid made of compressible and isotropic material could be modeled as a homogeneous body if it deforms homogeneously due to the application of the normal stresses on all of its six faces and the magnitude of the normal stresses on three orthogonal faces are different. A cuboid made of incompressible and isotropic material could be modeled as a homogeneous body, if it deforms homogeneously in two different biaxial experiments, such that the plane in which the forces are applied in the two biaxial experiments is mutually orthogonal.

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1. Introduction

Use of composites, metallic alloys, concrete, polymers which are believed to be inhomogeneous is on the raise. Apart from these man made materials, many naturally occurring bodies like arteries, tendons, ligaments, valves are also thought to be inhomogeneous. Because of the need to understand the mechanical response of these bodies, there is an ever growing literature devoted to understand the issues in the deformation of these bodies under applied loads. This study examines which bodies can be idealized as a homogeneous body when they are subjected to a non-dissipative mechanical process.

According to Truesdell and Noll (1965), when one is interested in purely mechanical processes, two material points $P_1, P_2 \in \mathcal{B}$ are said to be materially uniform, if there exist two placers κ_1 and κ_2 such that the neighborhoods $N_{\mathbf{X}_1}$ of $\mathbf{X}_1 = \kappa_1(P_1)$ and $N_{\mathbf{X}_2}$ of $\mathbf{X}_2 = \kappa_2(P_2)$ are indistinguishable with respect to their mechanical response. A body is said to be homogeneous if all the material points are materially uniform with respect to a single placement. A body that is not homogeneous is said to be inhomogeneous. This study focuses on one class of inhomogeneous bodies for which, the Cauchy stress, $\boldsymbol{\sigma}$ depends explicitly on the deformation gradient, $\mathbf{F}$ and the position vector of the material particle identified in the stress free reference configuration, $\mathbf{X}$, i.e., $\boldsymbol{\sigma} = \mathbf{g}(\mathbf{F}, \mathbf{X})$.

Many hold the opinion that the inhomogeneity of the type studied here could easily be decided by the body's response to electromagnetic radiation. They believe that if the body under investigation exhibits different responses in different regions as seen through, say, a microscope, it is inhomogeneous. However, different structures revealed under a microscope does not mean that the mathematical model of the body for its mechanical response should be different in these regions, if the mechanical properties and say, optical properties of the material are presumed[1] to be independent. Other reasons for the mathematical model for mechanical response could be different from that used for the response to electromagnetic radiation are explained below.

As inferred from its response to electromagnetic radiation all bodies are inhomogeneous at some scale. However, in case of bodies made of certain metals, say steel, having dimensions greater than a particular value seem to be robustly modeled using homogeneous models. Hence, it is believed that the homogeneous models are obtained through averaging the spatially varying

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[1] One cannot by knowing the refractive index, tell what the elastic properties of the material are.

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material parameters. These homogenization procedures are trying to replace a (spatially varying) function by a constant which cannot be robust unconditionally. In fact, for bodies undergoing large elastic deformations, Saravanan and Rajagopal (2003a,b, 2005, 2007) showed that the value of the constant material parameters in the homogeneous model so that the global load versus displacement relation is in agreement between the actual inhomogeneous body and its homogeneous counterpart, depends on the boundary value problem. Moreover, this constant material parameter varied by as much as 1800 percent with the boundary value problem used to determine these material parameters. This suggest that homogeneous models seem to work not because of homogenization but due to some other reason which could be that it is inherently homogeneous for its mechanical response under the investigated scenarios.

Taking the viewpoint that to answer questions of practical interest, such as, what is the maximum stress and displacement in a body subjected to some loading, the mathematical model for the body need not conform to the perceived reality that it is inhomogeneous, but can be an abstraction of the same. Akin to abstracting the earth as a point mass when one is interested in planetary motion, a rigid sphere when one is interested in studying eclipse we ask what would be a useful abstraction of a given body to capture some process that it is undergoing. Thus, in this point of view, an inhomogeneous model is required for a given body because some mechanical phenomena exhibited by this body can be captured only by abstracting it as an inhomogeneous body. In this spirit, the investigation here attempts on finding mechanical phenomena that requires a given body to be abstracted as an inhomogeneous body.

Towards this, in this article, we examine isotropic, inhomogeneous bodies, whose Cauchy stress depends explicitly on the left Cauchy–Green deformation tensor. It is assumed that the constitutive relation is a differentiable function of the position vector of material particles in the stress free reference configuration. On further assuming that this body undergoes a non-dissipative process from a stress free reference configuration, we examine scenarios when it would admit pure stretch homogeneous deformations when tested in the absence of any body forces. We find that a cuboid made of compressible and isotropic material can be considered to be homogeneous, if homogeneous[2] deformations are observed when the cuboid is subjected to normal stresses on all its six faces such that it does not result in a hydrostatic state of stress. A cuboid made of incompressible and isotropic material could be modeled as a homogeneous body, if homogeneous deformations are observed in two biaxial stretch experiments such that the plane in which the forces are applied is mutually orthogonal.

We emphasize that the above is a sufficient condition for abstracting a given body as a homogeneous body. On the other hand observing inhomogeneous deformations is only a necessary condition for the body to be inhomogeneous. Homogeneous body could also exhibit inhomogeneous deformations, because of the presence of body forces or non-uniform application of the boundary traction or due to the presence of inertial forces. Only on ruling out all these factors can the body be considered inhomogeneous. Thus, the proposed method seems to be a rationale way of deciding whether a given body can be idealized as homogeneous body or needs to be modeled as an inhomogeneous body.

Before proceeding further a few comments on the assumptions – isotropy and material functions being a differentiable function of the position vector – are necessary. First, we clarify that an inhomogeneous body can be made of isotropic constituents. The constitutive relation if for a point in the body and hence the material symmetry which restricts the form of this constitutive relation is also for a point. Inhomogeneity on the other hand is a statement about the form of the constitutive relation at different points. Since, point cannot have a structure there arises a conundrum as to the meaning of material symmetry. Thus, as even stated by Lekhnitskii (1981), the requirement that the symmetry of the constitutive relation be same as that of the material symmetry found based on the internal structure, is at best an assumption. Hence, it is advocated that one view material symmetry as a statement regarding the variation of the principal direction of the Cauchy stress with respect to the principal direction of the left Cauchy–Green deformation tensor. Paranjothi et al. (submitted for publication) presents experimental evidence and discusses practical difficulties associated with this view point. Consequently, a homogeneous body can be anisotropic and an inhomogeneous body can be made of isotropic constituents. This point that material symmetry and inhomogeneity are mutually exclusive cannot be overemphasized.

Next, the assumption that the material functions be differentiable function of the position vector needs discussion. Clearly, this assumption excludes bodies with voids, inclusions and the like. The results arrived here is applicable only for functionally graded materials. Relaxation of this assumption that the material functions be differentiable with respect to the position vector leads to mathematical complications and thereby obscuring the main thesis of this article that the idealization of a body as being homogeneous should be made based on the possibility of realizing homogeneous deformation field. Further, it is known (Varley and Cumberbatch, 1980; Ru et al., 2005) that a void or inclusion in a homogeneous matrix causes the deformation to be inhomogeneous when subjected to uniform far field loading. Therefore, it seems that scenarios when the deformation is homogeneous is more only for the case when the material functions are differentiable with respect to the position vector. However, a rigorous proof for the same is required and efforts are underway towards this. In Section 4, we briefly discuss how the result arrived at here could be used to study the case when the material functions are not differentiable with respect to position vector.

In the literature, it is prevalent to examine whether the body is subjected to homogeneous deformation. In fact, enormous care is taken to obtain homogeneous deformations, where possible. However, in most experiments only the surface deformation is measured. This surface measurement alone is not sufficient to determine if the realized deformation is homogeneous; deformation in the interior of the body also needs to be probed. On the other hand, if the surface deformation itself is non-uniform then the deformation is indeed inhomogeneous. There are reports of both the surface deformation being uniform (Rivlin and Saunders, 1951; Hariharaputhiran and Saravanan, 2010) and it being non-uniform (Kawamura et al., 1996; Lu et al., 2003; Paranjothi et al., 2011) in a pure stretch experiment showing the utility of the present approach to decide whether a given body can be approximated as homogeneous or otherwise. Further, X-ray computed tomography (Synolakis et al., 1996; Roux et al., 2008; You et al., 2009) as well as optical scanning tomography (Germaneau et al., 2007, 2008) techniques allows us to probe the deformation in the interior. As these techniques mature, the results in this paper would yield a practical tool for deciding when a body undergoing elastic deformations can be modeled as a homogeneous body.

One might think that the scale of observation would determine whether the deformation is homogeneous or not. This thinking stems from the observation that homogeneous deformation is seen in some bodies despite the fact that they are inhomogeneous at some length scale. However, mathematically a given deformation field would be either homogeneous or inhomogeneous with the

[2] If any straight line in the body deforms into another straight line the deformation is said to be homogeneous.

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