International Journal of Solids and Structures 50 (2013) 2817–2828

Contents lists available at SciVerse ScienceDirect

International Journal of Solids and Structures

journal homepage: www.elsevier.com/locate/ijsolstr

Effective spring stiffness for a periodic array of interacting coplanar penny-shaped cracks at an interface between two dissimilar isotropic materials

Huseyin Lekesiz [a,1], Noriko Katsube [a,*], Stanislav I. Rokhlin [b], Robert R. Seghi [c]

[a] Department of Mechanical Engineering, The Ohio State University, Columbus, OH, USA
[b] Department of Materials Science and Engineering, The Ohio State University, Columbus, OH, USA
[c] College of Dentistry, The Ohio State University, Columbus, OH, USA

ARTICLE INFO

Article history:
Received 30 August 2012
Received in revised form 21 January 2013
Available online 11 May 2013

Keywords:
Penny-shaped cracks
Dissimilar materials
Interface
Crack interaction
Spring stiffness
Ultrasound

ABSTRACT

An effective spring stiffness approximation is proposed for a hexagonal array of coplanar penny shaped cracks located at the interface between two dissimilar solids. The approximation is based on the factorization of the solution on the material dissimilarity factor, the crack interaction factor and the effective spring stiffness solution for non-interacting cracks in a homogeneous material. Such factorization is exact and was validated for 2D collinear cracks between two dissimilar solids. The crack interaction factor is obtained using a recently developed model for stress intensity factors for an array of coplanar penny shaped cracks in a homogeneous material; also the material dissimilarity function recently obtained for non-interacting penny shaped crack at the interface between two dissimilar materials is employed. The obtained solution is useful for an assessment by ultrasonic measurements of the interface stiffness in bonded structures for monitoring the interfacial microdamage growth due to mechanical loading and environmental factors.

1. Introduction

Bonded structures are widely used in various products and devices to improve structural durability and strength. The wide application of adhesive bonds in the aerospace industry for both aluminum and composite structures is well documented. In the field of dentistry, resin-retained ceramic restorations are performed to protect the remaining teeth and restore mechanical function without loss of aesthetics (Wang et al., 2007). Bonded interfaces are often compromised due to imperfect bonding conditions and degradation over time caused by various mechanical/thermal loadings and environmental factors. Micromechanical interfacial damage such as micro-cracks or micro-disbonds forms at the interfacial region and threatens the overall structural integrity.

Ultrasound methods are widely used to detect nondestructively different types of interfacial damage (Buck et al., 1989; Thompson and Thompson, 1991; Wang and Rokhlin, 1991; Margetan et al., 1992; Nagy, 1992; Rokhlin et al., 2004; Katoh et al., 2002; Milne et al., 2011). One approach in modeling elastic wave interaction with planar defects at the interface, such as micro-cracks or micro-disbonds, is to replace the microdefects-induced reduction in static stiffness by a continuous, uniform distribution of springs at the interface (Baik and Thompson, 1984; Sotiropoulos and Achenbach, 1988; Margetan et al., 1988; Lavrentyev and Rokhlin, 1994; Drinkwater et al., 1996; Delsanto and Scalerandi, 1998; Baltazar et al., 2003). This quasi-static approximation is demonstrated to be effective in modeling wave interactions at low frequencies, where the size of the damage is much smaller than the wavelength (Angel and Achenbach, 1985). The second approach is applicable when an interphase of finite thickness is formed because of material processing. This interphase has is its own constitutive properties affected by microdefects and if the microdefects are smaller than the interphase thickness their effect can be described by effective elastic properties (see for example Kachanov, 1994). Next, often when the wavelength is larger than the original layer thickness, an interphase between two media is replaced by an infinitely thin interface with appropriate boundary conditions, which is advantageous in solving the wave interaction problem (Rokhlin and Wang, 1991; Rokhlin and Huang, 1993; Hudson et al., 1997; Singher et al., 1994; Benveniste, 2006). For comparison of those two approaches see, for example, Lavrentyev and Rokhlin (1994) and Liu et al. (2000).

The objective of this work is to analyze the planar microdefects (microdisbonds) on an interface between two different solids. Such

* Corresponding author. Tel.: +1 614 292 0971; fax: +1 614 292 3163.
E-mail address: katsube.1@osu.edu (N. Katsube).
[1] Current address: Department of Mechanical Engineering, Bursa Technical University, Ataturk Kongre Kultur Merkezi, Merinos Parki, Osmangazi Bursa, Turkey.

0020-7683/$ - see front matter © 2013 Elsevier Ltd. All rights reserved.
http://dx.doi.org/10.1016/j.ijsolstr.2013.04.006

defects are modeled as an array of planar cracks, which are replaced by a continuum distribution of artificial springs at the interface, where the springs are selected in such a way that the stiffness reduction of the interface under far field loading is equivalent to that of the interface with an array of cracks. In order to be able to assess the bond integrity and the remaining life, accurate estimation of the percentage of disbond area (Palmer et al., 1988; Lavrentyev and Beals, 2000) is important. For this purpose, explicit expressions relating the spring stiffness constants to the crack geometry and density at the interface are desirable. For example, Baik and Thompson (1984) obtained the expression for effective normal spring stiffness for a planar array of periodically spaced strip cracks in a homogeneous material. They also obtained the corresponding expression for a single penny-shaped crack in a homogeneous material. Margetan et al. (1988) suggested an approximate expression for transverse spring stiffness for a single penny-shaped crack in a homogeneous material. Modifications of the Baik and Thompson (1984) and Margetan et al. (1988) methods were used by Lavrentyev and Rokhlin (1994) for an approximate description of an array of penny-shaped cracks between dissimilar materials in adhesive joints.

Most recently, Lekesiz et al. (2011a) obtained explicit analytical expressions for the normal and transverse effective spring stiffnesses of a planar periodic array of collinear cracks at an interface between two dissimilar isotropic materials based on the open crack model (Rice, 1988; Hutchinson and Suo, 1992). They (Lekesiz et al., 2011b) also obtained the normal and transverse spring stiffness expressions for a single penny-shaped crack at an interface between two dissimilar isotropic materials.

Most bonded interfaces consist of two dissimilar materials and micro-cracks or micro-disbonds at the interface are better represented by penny-shaped cracks than planar cracks. In addition, crack interactions play an important role in assessing the remaining life of bonded structures. Currently, however, there is no fracture mechanics based relationship between interfacial effective spring stiffness and crack density for interacting penny-shaped cracks at an interface between two dissimilar isotropic solids; such an approximate explicit relationship is obtained in this work.

2. Problem formulation for effective spring stiffness of cracked interface

The outline of our approach is presented schematically in Fig. 1. First, Fig. 1(a), the distributed effective interface spring stiffness is obtained to describe a periodic array of coplanar penny-shaped cracks between two identical elastic semispaces, and the effect of crack interactions on the spring stiffness is examined. For this, the array of planar cracks in the material is replaced by an artificial interface with a continuum distribution of springs utilizing our recent results (Lekesiz et al., 2013) where the effect of crack interactions on the stress intensity factors for a periodic array of coplanar penny-shaped cracks in a homogeneous material is obtained based on the approximate method by Kachanov and his co-workers (1985, 1987, 1989, 1994). Second, Fig. 1(b), the effect of material dissimilarity on the equivalent spring stiffness for a single (non-interacting) penny-shaped crack at an interface between two dissimilar materials is examined based on the work by Lekesiz et al. (2011b). Finally, similarly to the exact results by Lekesiz et al. (2011a) for an infinite array of 2D collinear cracks between two dissimilar solids, the effective spring stiffness is expressed in terms of three factors: crack interactions, material dissimilarity, and the spring stiffness of a single crack. Combining the crack interaction and material dissimilarity factors as shown in Fig. 1(a) and (b), respectively, and utilizing the effective spring stiffness for single penny shaped crack in a homogeneous material, we propose an approximate but explicit analytical expressions for equivalent spring stiffness for a periodic array of interacting penny-shaped cracks at an interface between two dissimilar materials as shown in Fig. 1(c).

The notion of the effective spring stiffness as depicted in Fig. 1 can be briefly described as follows. The far field displacement, Δ, can be separated into a displacement component without cracks, $\Delta_{no\text{-}crack}$, and an additional displacement due to the presence of cracks, Δ_{crack}, as follows.

$$\Delta = \Delta_{no-crack} + \Delta_{crack} \tag{1}$$

The idea (Baik and Thompson, 1984; Margetan et al., 1988; Lekesiz et al., 2011a) is to replace cracks by continuously distributed interfacial springs with the effective spring stiffness, k, such that they provide the same additional interface compliance (additional displacement Δ_{crack}) as that due to the cracks.

$$k_N = \frac{p^0}{\Delta_{N,crack}}, \quad k_T = \frac{t^0}{\Delta_{T,crack}} \tag{2}$$

where the subscripts N and T, respectively, denote the normal and transverse directions and p^0 and t^0, respectively, are the normal and shear traction applied at infinity.

The additional displacements can be determined using Castigliano's theorem, extended for cracked bodies (Tada et al., 2000), as in

$$\Delta_{N,crack} = \frac{\partial U_N}{\partial Q_N} \quad \Delta_{T,crack} = \frac{\partial U_T}{\partial Q_T} \tag{3}$$

where $Q_N = \pi b^2(p^0)$ and $Q_T = \pi b^2(t^0)$, respectively, represent the applied normal and transverse forces by considering the unit cylindrical cell with a circular cross sectional area $\pi\, b^2$ corresponding to the crack #1 region. The strain energies due to normal and shear tractions are respectively denoted by U_N and U_T. In obtaining the effective elastic spring stiffness for interacting coplanar penny-shaped cracks in a homogenous material in Eq. (2) (see Fig. 1(a)), we will first obtain the strain energy based on stress intensity factors, and then generalize this problem to the cracked interface between two solids.

3. Effective spring stiffness for a periodic array of interacting coplanar penny-shaped cracks at the interface between identical isotropic semispaces

3.1. Stress intensity factors

Consider a periodic array of coplanar penny-shaped cracks in an infinite medium subjected to remote normal and shear tractions as shown in Fig. 1(a). The hexagonal crack configurations (with crack radius a and crack periodicity b) as shown in Fig. 2(a) is considered. Lekesiz et al. (2013) numerically obtained the mode I, II and III stress intensity factors (SIFs), K_I, K_{II} and K_{III} for these interacting cracks as a function of crack density and the angle along the crack edge based on the approximate method developed by Kachanov and Laures (1989). The basic procedure for this analysis is summarized below.

The problem of N cracks subjected to remote tractions at infinity are replaced by equivalent problems where crack faces are loaded with normal and shear tractions, p^0 and t^0, respectively, and stresses vanishing at infinity. These equivalent problems with N cracks can be separated into N boundary value problems with each containing a single crack loaded by tractions which include crack interactions. Letting N go to infinity and using the fact that these N problems become identical, it is shown that the average traction for any crack (say crack #1) is magnified by a constant factor $\left(1 - \sum_{j=2}^{\infty} \Lambda_{j1}^{zz}\right)^{-1}$ due to crack interactions. The factor Λ_{j1}^{zz} is