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Progressive failure of a unidirectional fiber-reinforced composite using the method of cells: Discretization objective computational results

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ABSTRACT

The smeared crack band theory is implemented within the high-fidelity generalized method of cells micromechanics model to capture progressive failure within the constituents of a composite material while retaining objectivity with respect to the size of the discretization elements used in the model. Orientation of the crack band is determined using the maximum principal stress. When oriented perpendicular to the maximum principle stress the faces of the cracks in the crack band are subjected to only normal tractions and grow under pure mode I conditions. The traction–separation law governing the behavior of the crack band is related to the mode I fracture toughness, and formation of the crack band is initiated with a maximum stress criterion. Conversely, if the direction of the principal stress with the largest magnitude is compressive, it is assumed that the cracks within the crack band are constrained from growing in mode I. Instead, it is assumed that mode II cracks form within the crack band oriented along the plane of maximum shear stress. A Mohr–Coulomb initiation criterion is utilized to incorporate the effects of the normal tractions acting on the crack faces, and an effective shear traction is defined accordingly. The effective shear traction versus mode II separation law is a function of the mode II fracture toughness. A repeating unit cell containing 13 randomly arranged fibers is modeled and subjected to a combination of transverse tension/compression and transverse shear loading. The implementation is verified against experimental data and an equivalent finite element model that utilizes the same implementation of the crack band theory. Additionally, a sensitivity study is also performed on the effect of the size of the RUC on the stiffness and strength of the RUC.

1. Introduction

Micromechanics techniques can be employed to model the individual constituents within a composite material at the appropriate scale. Typically, a repeating unit cell (RUC) in the composite microstructure is identified and periodic boundary conditions are assumed. The response of a point in a continuum is determined assuming an infinite array of the RUCs. Additionally, representative volume element (RVE) methodologies exist which apply non-periodic boundary conditions to a subvolume that accurately represents the composite microstructure, and the size of the features of the microstructure is preserved. Micromechanics can be employed to provide the homogenized composite stiffness, or they can be used to model damage, failure, or other nonlinear phenomena locally within the constituents. If utilized for the latter, the global composite mechanisms arise through the natural evolution and interaction of the mechanisms in the constituents of the micromechanics model. Numerous micromechanical frameworks exist that encompass analytical, semi-analytical, and numerical techniques. An comprehensive review of micromechanics theories is given in Kanoute et al. (2009) and Aboudi et al. (2013).

In this work, the high-fidelity generalized method of cells (HFGMC) (Aboudi et al., 2001; Aboudi et al., 2013), is used to capture the nonlinear response, due to progressive failure, of a unidirectional fiber-reinforced polymer matrix composite (PMC) under a combination of transverse tension/compression and transverse shear loading. The focus is on developing a semi-analytical methodology that is insensitive to changes in the density of the numerical discretization, which is *necessary* for predicting damage state and failure. The generality of the HFGMC formulation admits any constitutive behavior at the subcell level. However if the response of the subcell material exhibits post-peak strain softening, the tangent stiffness tensor of the subcell loses positive definiteness. This leads to pathologically mesh dependent behavior. This mesh dependency, if not eliminated, will not provide predictive capability, rather, the models can be only used with confidence for “simulations”.

The smeared crack band approach (Bažant and Oh, 1983), which introduced a characteristic element length into the post-peak strain softening damage evolution formulation, can be used to

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alleviate the dependency of failure, and strain localization, on the discretization size within the continuum domain. The tangent slope of the softening stress–strain curve was scaled by the characteristic length to ensure that total strain energy release rate (SERR) upon complete failure (i.e., zero stress) is always equal to the prescribed fracture toughness (or critical SERR), regardless of the size of the elements. In the original formulation, the band was always oriented perpendicular to the direction of maximum principal stress; thus, the crack band always advanced under pure mode I. de Borst and Nauta (1985) and Rots and de Borst (1987) later reformulated the model to incorporate a fixed crack band that evolved under mixed-mode conditions. Both formulations employ triangular degradation schemes. Later, Camanho et al. (2007) incorporated more sophisticated initiation criteria, along with exponential degradation functions, to predict the onset of mixed-mode crack bands. All of these smeared crack formulations assume linear elastic behavior up to the initiation of the crack band, followed by immediate post-peak strain softening. However, Spencer (2002) coupled pre-peak plasticity with crack band governed post-peak strain softening to model failure of concrete. Recently, a thermodynamically-based work potential theory was developed which allows for pre-peak progressive damage (and hence nonlinearity in the pre-peak stress–strain response), as well as, post-peak progressive failure, governed by the smeared crack band approach, in homogenized laminae (Pineda and Waas, 2011), for fiber reinforced laminates.

A variation of the crack band theory (Bažant and Oh, 1983) is implemented here within HFGMC to model mesh objective failure of continuous fiber-reinforced PMCs. With this implementation, presented in Section 2, two scenarios are considered to determine the mode in which the cracks contained in the crack band grow. If the principal stress that has the highest magnitude is tensile, it is assumed that it is more energetically favorable for the crack band to form perpendicular to the maximum principal stress and for the cracks within the band to advance under mode I conditions. In this scenario, crack band initiation is indicated through the satisfaction of a maximum stress criterion, and the traction versus separation response of the crack band is governed by the mode I fracture toughness. Conversely, if the magnitude of a compressive principal stress is higher than the other principal stresses, the cracks within the crack band are assumed to be constrained from growing in mode I. Instead, it is assumed they evolve under mode II conditions (due to internal, Mohr–Coulomb friction) and are oriented with the plane of maximum shear stress. A Mohr–Coulomb initiation criterion is used to mark the formation of these shear crack bands, as well as, to define an effective shear traction that accounts the effect of the normal traction acting on the crack faces. The effective shear traction versus mode II separation is a function of the mode II fracture toughness.

Prior studies have utilized the finite element method (FEM) to model the composite microstructure. Plasticity was employed to model the non-linear response of the matrix and failure was introduced by allowing the fiber–matrix interface to debond using cohesive zone elements (Gonzaléz and Llorca, 2007; Totry et al., 2008b; Totry et al., 2010). The response of a statistical sample of RUCs subjected to a combination of transverse compressive and transverse shear was reported. Additionally, a multi-axial, mixed mode continuum damage theory was developed and implemented within HFGMC to model progressive failure of an RUC subjected to axial and transverse normal and shear loads (Bednarcyk et al., 2010).

In the current work, the focus is to evaluate the capabilities of the smeared crack band model to predict progressive failure evolution within a composite, at the microscale, using a semi-analytical method (HFGMC) by verifying and validating this implementation, as well as, the overall utility of the micromechanics model employed. Therefore, an analogous, fully numerical (FEM) model containing the same crack band implementation was used for a direct comparison. The objectivity of failure evolution with respect to the level of discretization used in HFGMC was previously demonstrated in Pineda et al. (2012).

In Section 3, the results for an RUC containing 13, randomly placed fibers subjected to a combination of transverse tension, transverse compression, and transverse shear are presented. The stress–strain response obtained from HFGMC is compared to the FEM results. Furthermore, failure path predictions obtained from the methods are analyzed. Finally, in Section 4 multiple realizations of RUCs containing 13 and 30 fibers are subjected to tension in order to probe the sensitivity of the stiffness and strength of the composite on the size of and fiber distribution within the RUC.

2. Modeling constituent-level post-peak strain softening with the smeared crack band approach

HFGMC is an efficient, semi-analytical tool useful for modeling microstructural details of a composite material (Aboudi et al., 2001). With HFGMC, a composite RUC is discretized into any number of parallelpiped subvolumes, called subcells, see Fig. 1. The subcells can occupied by any material yielding a composite with any number of phases. Quadratic displacement fields are assumed in each of the subcells; the generalized method of cells (GMC) incorporates linear displacement approximations. Displacement and traction continuity conditions are enforced, in an average integral sense, at the subcell interfaces, along with periodic boundary conditions at the RUC boundaries. In addition, the zeroth, first, and second moments of equilibrium are utilized to derive a set of equations that yield strain concentration matrix which relates the local subcell strains to the global composite strains. Following determination of the subcell strains, the subcell stresses are readily calculated using the local constitutive laws, and volume averaging can be used to obtain the homogenized thermomechanical properties of the composite. With HFGMC (and GMC) stress concentrations can be resolved at the microscale. As opposed to mean field theories, which simply provide a single matrix stress and a single fiber stress (Eshelby, 1957; Hashin, 1962; Christensen and Waals, 1972; Mori and Tanaka, 1973). HFGMC was reformulated to improve the computational efficiency, and the reader is referred to Aboudi et al. (2013) for the complete details. This semi-analytical method offers an efficient tool for computing local fields within a composite RUC and is amenable for implementation in a multiscale framework.

Physics-based, discretization objective, constitutive theories for modeling progressive failure must be in place to accurately predict the response of a structure that is failing. For pre-peak loading (i.e., positive-definite tangent stiffness tensor), there are a multitude of non-linear elastic, plastic, continuum damage mechanics, and viso-elastic/plastic theories available that can predict the evolution of the appropriate mechanisms in the composite. However, when the local fields enter the post-peak regime of the stress–strain laws, most of these theories breakdown in a numerical setting and display pathological mesh dependence (Bažant and Cedolin, 1979; Pietruszczak and Mroz, 1981). Loss of positive-definiteness of the tangent stiffness tensor leads to a material instability, which manifests as a localization of damage into the smallest length scale in the continuum problem (Bažant and Cedolin, 1991). In HFGMC this is a single subcell. Thus, the post-peak softening strain energy is dissipated over the volume of the subcell that the damage localizes to. Since a stress–strain relationship prescribes the energy density dissipated during the failure process, the total amount of energy dissipated in the subcell is proportional to the size of the subcell, and in the limit as the subcell size is decreased, zero energy is required to fail the structure.