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Steady-state solutions of a propagating borehole

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ABSTRACT

This paper analyzes a general class of stationary trajectories for deep boreholes drilled using rotary systems. These solutions correspond to helical wells twisting around a vertical axis, which can degenerate into straight or circular boreholes; they arise when the forces acting on the bit, and thus the penetrations of the bit into the rock, are invariant in a basis attached to the bit. Under these stationary conditions, the deformed configuration of the bottomhole assembly (the lower part of the drillstring) is also invariant. The paper formulates the equations governing these equilibrium solutions from considerations involving the interaction between the bit and the rock, but also between the bottomhole assembly and the borehole through the contact points at the stabilizers and at the rotary steerable system (the tool used to steer the bit). It is shown that the stationary solutions are completely defined by four parameters characterizing the geometry of the wellbore: two at the scale of the bottomhole assembly (the curvature and inclination of the helical axis), and two at the scale of the bit (the bit tilts, proxies for the borehole diameter). The key dimensionless parameters that control the directional response of the drilling system are finally identified, as well as the critical values of some parameters at which a pathological change in the response takes place.

1. Introduction

Drilling deep boreholes that weave complex trajectories in the subsurface has been made possible with the development of directional drilling systems. The need for complicated well paths arises, within the context of exploration and production of oil and gas, from a variety of reasons that include: accessing a hydrocarbon reservoir capped by a salt dome, complying with constraints on the location of the rig relative to the target, drilling multiple wells from one location to reduce cost or limit the environmental impacts, and rescuing a distressed well. A key factor in the ability to construct complex wells has been the emergence of the *rotary steerable systems* (RSS) in the late 1990's. These servo-controlled downhole robots steer the bit by either applying a force on the borehole wall or by tilting the bit.

Fig. 1 sketches a rotary drilling structure used in the oil and gas industry. This structure comprises a rig, from which is suspended the *drillstring,* a hollow slender tube that can reach several kilometers in length. The rotary speed and the axial force (*hookload*) are imposed at the rig. The lower part of the drillstring is the *bottomhole assembly* (BHA), which is usually about a hundred meters long. It consists of heavier pipes and short elements of larger diameter, called *stabilizers*, which center the BHA in the borehole (Inglis, 1987). The bit is at the lower extremity of the BHA. While the main part of the drillstring is in tension under its own weight, the BHA or part of it is in compression in order to induce a sufficient *weight on bit*, the axial force transmitted to the bit.

The RSS is located close to the bit, between the bit and the first stabilizer. Such semi-automated device works in association with sensors and downhole control units in order to steer the borehole (Downton et al., 2000). Considerations are restricted here to *push-the-bit* RSS, which use a set of extensible pads to apply a lateral force on the borehole wall at a designated location.

Once stripped of all its mechanical components, a drilling structure is, in its simplest abstraction, an extremely slender elastic body constrained to deform inside the borehole. It is subject to gravity, hydraulic forces associated to the flow of drilling mud, imposed forces at the rig and at the RSS, forces and moments at the bit when drilling, and reaction forces at its contacts with the borehole. The directional propagation of the borehole depends on the forces and moments at the bit; they are affected not only by the action of the RSS but also by the deformed configuration of the BHA, which is constrained by the stabilizers to espouse the existing borehole. The propagation of the borehole is thus governed by the interaction between a geometrical object, the wellbore, and a mechanical object, the drilling structure.

During drilling, the evolving borehole is described at the scale of the BHA as a propagating 3D curve, but also by its varying cross

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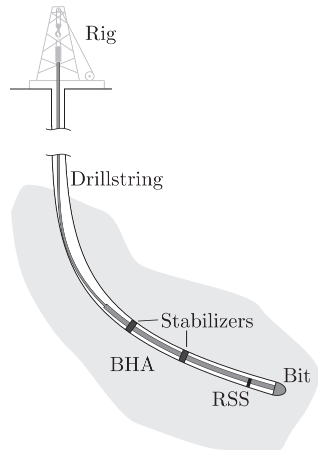

Fig. 1. Sketch of a typical directional drilling apparatus equipped with a push-the-bit RSS. The number of stabilizers is usually greater than 2.

section when viewed at the scale of the bit. The overgauge of the borehole with respect to the bit diameter is caused in part by the tilt of the bit on the borehole. Hence, the borehole can conveniently be described by four functions of a curvilinear coordinate running along its axis: the inclination and azimuth of its axis to describe the geometry of the borehole viewed as a curve, and two tilt angles that characterize its geometry at the scale of the bit.

Study of this dynamical system is motivated by engineering issues, which include defining the criteria of bit selection, designing the BHA, or devising an appropriate controller for the RSS with the objective of tracking a predefined well path. The starting point is to formulate a mathematical model of the propagating borehole, i.e., to derive the equations governing the spatial evolution of the inclination and azimuth of the borehole, and of the tilt angles of the bit. These equations must embed a description of the mechanical system that creates the borehole.

Only a handful of contributions dealing with the formulation of such a mathematical model can be found, however (Neubert and Heisig, 1996; Downton, 2007; Detournay and Perneder, 2011; Perneder and Detournay, 2013a). Moreover, these efforts are restricted to particular cases. On the other hand, many numerical solutions without any explicit description of the underlying mathematical model have been proposed (Callas, 1981; Millheim, 1982; Brett et al., 1986; Rafie, 1988; Maouche, 1999; Studer et al., 2007). Related efforts include those aimed at understanding the mechanics of the BHA constrained to deform within the borehole (Murphey and Cheatham, 1966; Fischer, 1974; Bradley, 1975; Millheim et al., 1978; Ho, 1986; Rafie et al., 1986; Birades and Fenoul, 1988; Jogi et al., 1988; Dogay et al., 2009) and at deriving the interface laws that govern the interaction of the bit with the rock formation (Cheatham and Ho, 1981; Ho, 1987; Ho, 1995; Menand et al., 2002; Detournay et al., 2008; Franca, 2010; Perneder et al., 2012).

Despite the lack of a comprehensive formulation of the equations governing the borehole propagation, the equilibrium points of this dynamical system can still be derived somewhat independently. Before clarifying the nature of these stationary solutions, a word of explanation is needed to justify their existence in the presence of a lengthening borehole and thus an evolving mechanical system. The arguments rely on recognizing the appropriate scale for the borehole propagation model and on accounting for the nature of the equations that govern the deformation of the BHA.

First, three length scales can naturally be identified in this problem. One corresponds to the dimensions of the bit, typically of order $O(0.1\text{ m})$. The bit is viewed as a three-dimensional object at this length scale, which is thus used when deriving interaction laws between the bit and the rock formation. The second length scale, of order $O(1 \sim 10\text{ m})$, is associated with the dimensions of the BHA; more precisely, with the distance between successive contact points between the BHA and the borehole. Finally, the third one, of order $O(10^3\text{ m})$, is related to the problem of the entire drillstring. As the borehole is propagating, the drillstring becomes longer but the BHA length remains the same.

Second, the propagation of the borehole is predominantly affected by the positions along the BHA of the stabilizers closest to the bit, all other parameters being fixed. Indeed, the first n stabilizers mainly control the steering of the bit in the sense that altering the composition of the BHA by removing or repositioning the $(n+1)$th stabilizer, while keeping the first n unchanged, hardly affects the directional response of the system. (For most practical purposes, $n = 3$ is sufficient.) The weakening influence of an additional stabilizer derives from the nature of the equations governing the deformation of the BHA. This implies that it is appropriate to "cut" the BHA above the nth stabilizer and replace the rest of the drillstring by forces and moments, which themselves are supposed unaffected by the steering of the bit and by the RSS force. The forces and moments at the cut can be calculated using a drillstring model, also called *torque and drag* model, which is mainly concerned with the transmission of the axial force and torque along the drillstring (Johancsik et al., 1984; Ho, 1988; Aadnoy et al., 1998; Menand et al., 2006; Denoël and Detournay, 2011). Here the challenge is to identify the contacts between the drillstring and the borehole.

In summary, a borehole propagation model can be constructed at the intermediate scale, i.e., at the BHA scale, with boundary conditions capturing the processes at the two other scales. At the lower boundary, the bit is collapsed onto a point, with the details of the bit/rock interaction encapsulated into interface laws consisting in relationships between the forces and moments at the bit and the conjugated kinematic quantities (Detournay et al., 2008; Perneder et al., 2012). At the upper boundary, the action of the drillstring is replaced by assumed known forces, with the axial force being usually the only non-zero component.

All the dynamic and kinematic variables that are introduced in the borehole propagation model are quantities averaged over many revolutions. Indeed, the time scale associated with directional drilling is considerably larger than the period of revolution, itself larger or of the same order as the period of vibrations of the bit and drillstring. Moreover, with the bit collapsed to a point, the spatial resolution of the model cannot be less than the bit size, itself much larger than the bit penetration per revolution. This averaging thus implies that the borehole propagation can be viewed as a quasi-static process, with the dynamic effects possibly subsumed into parameters of the model. For example, the effects of bit whirl could be accounted for as an overgauging of the borehole.

By restricting the model to the intermediate scale and in view of the quasi-static nature of the propagation, it is indeed possible to contemplate the existence of stationary solutions. They correspond to situations, for which all the forces acting on the BHA remain unchanged in a reference system attached to the BHA (from which the angular velocity has been abstracted, however), as the bit propagates the hole. The existence of such solutions evidently requires that the rock has homogeneous and isotropic properties.

This force invariance has two implications. First, the suitably averaged motion of the BHA axis appears to be that of a rigid body, as the averaged deformed configuration of the BHA remains unchanged. Second, the penetration variables, which are associated with the advancement of the borehole, are stationary. The only solutions verifying these stationary conditions are in fact helical boreholes winding around a vertical axis. These solutions can degenerate, however, into straight inclined boreholes and circular horizontal boreholes.