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Mechanical modeling of helical structures accounting for translational invariance. Part 1: Static behavior

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ABSTRACT

The purpose of this paper is to investigate the static behavior of helical structures under axial loads. Taking into account their translational invariance, the homogenization theory is applied. This approach, based on asymptotic expansion, gives the first-order approximation of the 3D elasticity problem from the solution of a 2D microscopic problem posed on the cross-section and a 1D macroscopic problem, which turns out to be a Navier–Bernoulli–Saint-Venant beam problem. By contrast with earlier references in which a reduced 3D model was built on a slice of the helical structure, the contribution of this paper is to propose a 2D microscopic model. Homogenization is first applied to helical single wire structures, i.e. helical springs. Next, axial elastic properties of a seven-wire strand are computed. The approach is validated through comparison with reference results: analytical solution for helical single wire structures and 3D detailed finite element solution for seven-wire strands.

1. Introduction

Helical structures are widely used in mechanical and civil engineering applications. These structures are usually subjected to large loads which can lead to the material degradation and cracks associated with corrosion and mechanical fatigue. This threatens the structural strength. In this framework, non-destructive testing is a crucial tool for detection, localization and measurement of material discontinuities. The choice of the appropriate technique depends on dimensions and accessibility of the structure. Particularly, ultrasonics allow to control large components, such as plates and tubes, by analyzing their elastic guided waves. The purpose of this study, which is composed of two parts, is to develop a numerical model for the analysis of the elastic wave propagation phenomenon in prestressed helical structures. This problem requires the computation of the static prestress state. Therefore, a first model will be developed in Part 1 of this paper, to compute this static state. Taking into account this prestress state, a second model will be developed in Part 2, in order to analyze the wave propagation in these prestressed structures. The goal of this first part of this paper is thus to develop an approach that allows the computation of the prestress state in helical structures subjected to axial load.

Numerous works have been devoted to the modeling of the static behavior of helical structures as springs and multi-wire cables under axial loads. For helical springs, an analytical model was proposed among others in Ancker and Goodier (1958) and Wahl (1963) considering the spring as an Euler–Bernoulli beam with pitch and curvature corrections. Numerical approaches describing the static behavior of helical springs have been also developed. Among these works, a finite element model of half of a spring slice has been proposed in Jiang and Henshall (2000).

The static behavior of seven-wire strands has been widely studied in literature. Various analytical models based on different assumptions have been proposed, such as the model of Costello (1977) which is one of the most popular. These models are reviewed in Jolicoeur and Cardou (1991) and compared in Jolicoeur and Cardou (1991) and Ghoreishi et al. (2007). Besides, numerical models relying on the finite element method were developed. Some of them are based on beam elements (Durville, 1998; Nawrocki and Labrosse, 2000; Páczelt and Beleznai, 2011), see also Nemov et al. (2010) and Bajas et al. (2010) in which ITER superconducting cables composed of a large number of strands are studied. But most of the time, 3D models are used, see e.g. Boso et al. (2006), Ghoreishi et al. (2007), İmrak and Erdönmez (2010), Nemov et al. (2010), Stanova et al. (2011a,b) and Erdönmez and İmrak (2011). In order to obtain a good representation of the geometry as well as the displacement solution, which may involve bending phenomena, quadratic elements are employed. This leads to models which can be computationally expensive, when the model axial length is about the pitch length. Therefore, as soon as the loading fulfills helical symmetry, one can take benefit of this property to reduce the model size. This has been achieved in Jiang et al.

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(1999) and Jiang et al. (2008) in which the computational domain is restricted to a basic sector of a helical slice. Helical symmetry may also be accounted for within the framework of homogenization theory. This has been proposed first in Cartraud and Messager (2006) using axial periodicity, and then improved in Messager and Cartraud (2008), in which helical symmetry enables to consider one slice of a strand. The derivation of the slice model is different in Jiang et al. (1999), Jiang et al. (2008) and Messager and Cartraud (2008). However, in both cases, helical symmetry yields displacement constraints between the two faces of the slice, with a loading under the form of an axial strain and a twist rate.

This work further advances Cartraud and Messager (2006) and Messager and Cartraud (2008), taking advantage of the translational invariance. Helical symmetry can be actually considered more efficiently. Thus the model can be reduced to a 2D one, i.e. a cross-section model. This requires to formulate the homogenization theory in a twisted coordinate system. This technique then allows the computation of the static prestressed state of helical structures (single wire and multi-wire) from the solution of a 2D problem. Let us mention that an advanced analytical 2D model has been recently proposed in Argatov (2011). This model takes into account Poisson's effect, contact deformation and allows to obtain the overall strand stiffness as well as local contact stresses. In this reference, plane strain was assumed to formulate the 2D problem while in the present work helical symmetry is used.

The method developed in this paper is restricted to multi-wire helical structures composed of a stack of helical wires wrapped with the same twisting rate around a straight axis. As explained in Section 3, this excludes the case of double helical structures (such as independent wire rope core for instance) and cross-lay strands.

This paper is organized as follows. First, in Section 2, the curvilinear coordinate system is introduced. Then in Section 3 the translational invariance is defined, which is a necessary condition for the helical homogenization approach. Based on the asymptotic expansion method and exploiting the translational invariance property, the homogenization procedure is presented in Section 4. Its finite element solution is detailed in Section 5. The helical homogenization approach is validated for helical single wire and seven-wire structures by comparison with analytical or numerical models in Section 6.

2. Curvilinear coordinate system

A helical structure is considered (see Fig. 1). Let $(\mathbf{e}_X, \mathbf{e}_Y, \mathbf{e}_Z)$ its Cartesian orthonormal basis. The helix centreline is defined by its helix radius R in the Cartesian plane $(\mathbf{e}_X, \mathbf{e}_Y)$ and the length of one helix pitch along the Z-axis denoted by L. This helix centerline can be described by the following position vector:

$$\mathbf{r}(s) = R\cos\left(\frac{2\pi}{l}s+\theta\right)\mathbf{e}_X + R\sin\left(\frac{2\pi}{l}s+\theta\right)\mathbf{e}_Y + \frac{L}{l}s\mathbf{e}_Z, \qquad (1)$$

where $l = \sqrt{L^2+4\pi^2R^2}$ is the curvilinear length of one helix pitch and θ is the helix phase angle in the $Z=0$ plane. For a seven-wire strand, θ is equal to $(N-1)\pi/3$, where $N = 1,..,6$ refers to the number of the helical wire. θ is equal to zero for a single wire helical structure. The helix lay angle Φ is defined by $\tan\Phi = 2\pi R/L$. A complete helix is described by the parameter s varying from 0 to l.

2.1. Serret–Frenet basis

A Serret–Frenet basis $(\mathbf{e}_n, \mathbf{e}_b, \mathbf{e}_t)$ associated to the helix can be defined (see e.g. Gray et al. (2006)), where the unit vectors $\mathbf{e}_n, \mathbf{e}_b, \mathbf{e}_t$ are given by $\mathbf{e}_t = d\mathbf{r}/ds, d\mathbf{e}_n/ds = \tau\mathbf{e}_b - \kappa\mathbf{e}_t$ and $d\mathbf{e}_b/ds = -\tau\mathbf{e}_n$. For helical curves, the curvature $\kappa = 4\pi^2R/l^2$ and

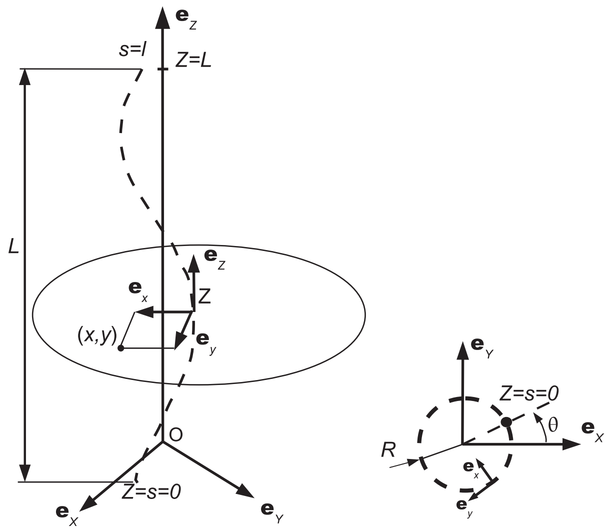

Fig. 1. Left: One helix pitch and its twisted basis associated to the twisted coordinate system (x, y, Z). Right: view normal to the Z-axis. The point $Z = s = 0$ lies in the $(\mathbf{e}_X, \mathbf{e}_Y)$ plane.

the torsion $\tau = 2\pi L/l^2$ are constant. In the Cartesian basis, $\mathbf{e}_n, \mathbf{e}_b$ and $\mathbf{e}_t$ are expressed by:

$$\begin{aligned}
\mathbf{e}_n &= -\cos\left(\frac{2\pi}{l}s+\theta\right)\mathbf{e}_X - \sin\left(\frac{2\pi}{l}s+\theta\right)\mathbf{e}_Y,\\
\mathbf{e}_b &= \frac{L}{l}\sin\left(\frac{2\pi}{l}s+\theta\right)\mathbf{e}_X - \frac{L}{l}\cos\left(\frac{2\pi}{l}s+\theta\right)\mathbf{e}_Y + \frac{2\pi}{l}R\mathbf{e}_Z,\\
\mathbf{e}_t &= -\frac{2\pi R}{l}\sin\left(\frac{2\pi}{l}s+\theta\right)\mathbf{e}_X + \frac{2\pi R}{l}\cos\left(\frac{2\pi}{l}s+\theta\right)\mathbf{e}_Y + \frac{L}{l}\mathbf{e}_Z.
\end{aligned} \qquad (2)$$

The normal vector $\mathbf{e}_n$ remains parallel to the $(\mathbf{e}_X, \mathbf{e}_Y)$ plane while $\mathbf{e}_b$ and $\mathbf{e}_t$ move in the three directions of the Cartesian basis as s and θ vary.

2.2. Twisted basis

A special case of the Serret–Frenet basis denoted by $(\mathbf{e}_x, \mathbf{e}_y, \mathbf{e}_Z)$ corresponding to $\kappa = 0$ and $\tau = 2\pi/L$ can be considered. It corresponds to a twisted coordinate system along the Z-axis ($s \equiv Z$) with axial periodicity L. The unit vectors $\mathbf{e}_x$ and $\mathbf{e}_y$ rotate around the Z-axis and remain parallel to the $(\mathbf{e}_X, \mathbf{e}_Y)$ plane (see Fig. 1). In the Cartesian basis, $\mathbf{e}_x$ and $\mathbf{e}_y$ are expressed as:

$$\begin{aligned}
\mathbf{e}_x &= -\cos\left(\frac{2\pi}{L}Z+\theta\right)\mathbf{e}_X - \sin\left(\frac{2\pi}{L}Z+\theta\right)\mathbf{e}_Y,\\
\mathbf{e}_y &= \sin\left(\frac{2\pi}{L}Z+\theta\right)\mathbf{e}_X - \cos\left(\frac{2\pi}{L}Z+\theta\right)\mathbf{e}_Y.
\end{aligned} \qquad (3)$$

It should also be noted that this twisted coordinate system coincides with the one proposed in Onipede and Dong (1996), Nicolet et al. (2004) and Nicolet and Zola (2007) for the analysis of twisted and helical structures.

2.3. Covariant and contravariant bases

Differential operators can not be expressed directly in the Serret–Frenet or twisted bases. They have first to be expressed in the covariant and contravariant bases. The reader can find an in-depth treatment of curvilinear coordinate systems in Chapelle and Bathe (2003); Synge and Schild (1978); Wempner (1981) for instance.

From the twisted basis $(\mathbf{e}_x, \mathbf{e}_y, \mathbf{e}_Z)$, a new coordinate system (x, y, Z) is built, for which any position vector can be expressed as: