International Journal of Solids and Structures 50 (2013) 1383–1393

Contents lists available at SciVerse ScienceDirect

International Journal of Solids and Structures

journal homepage: www.elsevier.com/locate/ijsolstr

Mechanical modeling of helical structures accounting for translational invariance. Part 2 : Guided wave propagation under axial loads

Fabien Treyssède [a,*], Ahmed Frikha [a], Patrice Cartraud [b]

[a] LUNAM Université, IFSTTAR, MACS, F-44344 Bouguenais, France
[b] LUNAM Université, GeM, UMR CNRS 6183, Ecole Centrale de Nantes, 1 rue de la Noë, 44 321 Nantes Cédex 3, France

ARTICLE INFO

Article history:
Available online 21 January 2013

Keywords:
Waveguide
Prestress
Helical coordinates
Finite element
Strands
Springs

ABSTRACT

This paper corresponds to the second part of a study that aims at modeling helical structures accounting for translational invariance. In the Part 1 of this paper, the static behavior has been addressed using a helical homogenization approach which provides the stress state corresponding to axial loads. The latter is considered as a prestressed state, for elastic wave propagation analysis in helical waveguides, which is the subject of the Part 2 of this paper. Non destructive testing of springs and multi-wire strands is a potential application of the proposed model. Accounting for translational invariance, the elastodynamic equations of prestressed helical structures yield a 2D problem posed on the cross-section, corresponding to a so-called semi-analytical finite element (SAFE) formulation. For helical springs, the numerical model is validated with an analytical solution corresponding to a Timoshenko beam approximation. It is shown that the influence of the prestressed state is significant at low frequencies. Finally, a seven-wire strand subjected to axial loads is considered. The computed dispersion curves are compared to experimental data. Good agreement is obtained for the first compressional-like modes and their veering central frequency.

1. Introduction

This paper is the second part of a study that aims at modeling helical structures accounting for translational invariance. In Part 1, the static state in the case of axial loads has been addressed. Taking into account the effects of prestress and geometry deformation due to these static loads, the objective of Part 2 is the computation of wave modes guided by the helical structures.

Inspection methods based on elastic guided waves are among the most popular techniques of non destructive testing. Due to the complexity of signals, this technique is often restricted to simple geometries such as plates and pipes. The computation of modes of propagation in more complex geometries (arbitrary cross-section, curved axis,...) requires appropriate simulation tools, typically based on finite element methods.

A first method based on the Floquet conditions, applicable to periodic structures, has been used for straight structures (Gry and Gontier, 1997; Duhamel et al., 2006; Mencik and Ichchou, 2007) and for helical waveguides (Treyssède, 2007). A more efficient method, valid for translationally invariant structures and often referred to as the semi-analytical finite element (SAFE) method, has also been developed. This technique has been proposed in early works in Dong and Nelson (1972). With this method, the problem is reduced on the cross-section, which decreases the computation time. More recently, the SAFE method has been used for straight waveguides with arbitrary cross-section (Gavric, 1995; Damljanovic and Weaver, 2004; Hayashi et al., 2006; Jezzine, 2006) or material complexity (Rattanawangcharoen et al., 1992; Zhuang et al., 1999; Bartoli et al., 2006; Marzani, 2008). This approach has also been applied to curved waveguides: twisted in Onipede and Dong (1996), toroidal in Demma et al. (2005) and Finnveden and Fraggstedt (2008) and helical in Treyssède (2008). Finally, a SAFE method modeling the propagation of elastic waves in seven-wire strands has been developed in Treyssède and Laguerre (2010).

Helical structures such as springs and strands are generally subjected to axial loads. The above-mentioned works are restricted to the propagation of guided waves in unloaded structures. Only few studies have extended the SAFE method to loaded waveguides. Straight waveguides under axial loads have been considered in Chen and Wilcox (2007) and Loveday (2009). To the authors knowledge, there is no general model in the literature that allows to determine guided modes propagating in prestressed curved waveguides.

Therefore the goal of this paper is to propose a numerical model for the propagation of guided waves in helical structures subjected to axial loads, particularly in prestressed multi-wire strands. This

* Corresponding author.
E-mail address: fabien.treyssede@ifsttar.fr (F. Treyssède).

0020-7683/$ - see front matter © 2013 Elsevier Ltd. All rights reserved.
http://dx.doi.org/10.1016/j.ijsolstr.2013.01.006

study is limited to linear elastic materials. The SAFE method is adopted, which allows to solve the 3D elastodynamic equations of motion thanks to a 2D model and without beam approximation.

The method developed in this paper is restricted to multi-wire helical structures composed of a stack of helical wires wrapped with the same twisting rate around a straight axis. As explained in Section 3 of Part 1, this excludes the case of double helical structures (such as independent wire rope core for instance) and cross-lay strands.

The paper is organized as follows. Considering the static state computed in Part 1 as the prestressed state, the variational formulation associated with the superimposed linear dynamics is first described in Section 2. The twisting coordinate system is then introduced and differential operators are expressed in this system in Section 3. Exploiting the translational invariance property, the 3D variational formulation is then reduced in Section 4 to a 2D problem posed on the cross-section, which is classical in the framework of SAFE methods. In Section 5, an energy velocity expression is derived for prestressed waveguides. Using SAFE matrices, the equality between group and energy velocities is proved for undamped materials. Then for helical springs, numerical results are compared in Section 6 to those of a beam model proposed in Frikha et al. (2011). For seven-wire strands subjected to axial loads, using stick contact conditions between the core and peripheral wires, numerical results are compared to experimental data in Section 7.

2. Dynamic motion of prestressed structures

The analysis of the dynamics of prestressed structures can be decomposed into a static problem, solved in Part 1 of this paper, and the motion superimposed on this prestressed state, which is the aim of Part 2. Therefore, three configurations must be distinguished: the initial configuration (without initial stress), the prestressed static configuration (which is denoted V_0) and the final configuration including dynamics. An updated Lagrangian formulation is used, the variables being expressed in the prestressed static configuration.

One assumes a linear and elastic material behavior and a time-harmonic $e^{-i\omega t}$ evolution of the solution. Considering small-amplitude waves as perturbations onto the prestressed static state, the 3D variational formulation governing elastodynamics is given by (see e.g. Bathe (1996) and Yang and Kuo (1994)):

$$\forall \delta\mathbf{u}, \quad \int_{V_0} \delta\boldsymbol{\epsilon} : \mathbf{C}_0 : \boldsymbol{\epsilon}\, dV_0 + \int_{V_0} \mathrm{tr}(\nabla_0 \delta\mathbf{u} \cdot \boldsymbol{\sigma}_0 \cdot \nabla_0 \mathbf{u}^T) dV_0 - \omega^2 \int_{V_0} \rho_0 \delta\mathbf{u} \cdot \mathbf{u}\, dV_0 = 0, \tag{1}$$

with $\delta\mathbf{u}$ kinematically admissible and where $\mathbf{u}$ and $\boldsymbol{\epsilon} = 1/2(\nabla_0\mathbf{u} + \nabla_0\mathbf{u}^T)$ denote the displacement and the strain tensor, respectively. The subscript 0 refer to the prestressed static configuration: $\mathbf{C}_0, \rho_0$ and V_0 denote the elasticity tensor, the material density and the structural volume in the prestressed configuration. $\mathrm{tr}(\cdot)$ is the trace and ∇_0 is the gradient operator with respect to the prestressed configuration. $\boldsymbol{\sigma}_0$ is the Cauchy prestress, i.e. the stress tensor associated with the prestressed state. The second term of the formulation, related to $\boldsymbol{\sigma}_0$, is sometimes referred to as the geometric stiffness in the literature.

In the context of non-linear mechanics, Eq. (1) is the so-called linearized updated Lagrangian formulation, representing the motion of small perturbations superimposed on a given state. Its derivation requires a non-linear geometrical analysis (large displacement or strain). This implies that the prestressed configuration should correspond to a non-linear geometrical state. Yet in this paper, one will assume that the effects of non-linearity of the prestressed state can be neglected on dynamics, and the linear computations of Part 1 will be used for simplicity.

3. Formulation in the curvilinear coordinate system

For the wave propagation analysis in curved waveguides, the variational formulation described in Section 2 must be expressed in an appropriate curvilinear coordinate system. In this paper, a coordinate system that satisfies translational invariance both for helical single-wire and multi-wire waveguides is required. Therefore, the twisted basis is chosen. The translational invariance property will be checked in Section 4. The reader may refer to Part 1 of this paper for more details.

3.1. Twisted basis

One considers a helical single-wire waveguide (see Fig. 1 in Part 1). Let $(\mathbf{e}_X, \mathbf{e}_Y, \mathbf{e}_Z)$ denotes the Cartesian orthonormal basis. The centreline is defined by a helix of radius R in the Cartesian plane $(\mathbf{e}_X, \mathbf{e}_Y)$ and pitch L along the Z-axis. The helix lay angle Φ is defined by $\tan\Phi = 2\pi R/L$.

The twisted basis $(\mathbf{e}_x, \mathbf{e}_y, \mathbf{e}_Z)$ has been defined in Part 1, as an orthonormal basis rotating around the Z-axis. It corresponds to a particular case of helical system with $\kappa = 0$ and $\tau = 2\pi/L$, where κ and τ denote the curvature and the torsion respectively. The unit vectors $\mathbf{e}_x$ and $\mathbf{e}_y$ are expressed in the Cartesian basis by Eq. (3) of Part 1.

However throughout Part 2, geometrical parameters R, L, Φ, κ and τ are now associated with the prestressed configuration, i.e. the deformed helix under the action of the static axial load. Rigorously, these parameters should be denoted with subscripts 0, omitted for brevity's sake of notations throughout Part 2. When needed, we will use subscripts i (R_i or Φ_i for instance) to refer to the initial geometrical parameters, i.e. parameters associated with the initial configuration (without initial stress).

In order to express differential operators in the twisted basis, one has to develop them in the covariant and contravariant bases, $(\mathbf{g}_1, \mathbf{g}_2, \mathbf{g}_3)$ and $(\mathbf{g}^1, \mathbf{g}^2, \mathbf{g}^3)$, which have been defined by Eqs. (5) and (7) in Part 1.

One recalls that the Christoffel symbol of the second kind Γ^k_{ij} can be calculated from $\Gamma^k_{ij} = \mathbf{g}_{i,j} \cdot \mathbf{g}^k$, where $\mathbf{g}_{i,j}$ corresponds to the derivatives of the covariant basis. Its expression in the twisted basis has been obtained in Eq. (8) of Part 1.

As a side remark, note that twisting coordinates have also been used for elastic wave propagation in pretwisted beams (Onipede and Dong, 1996), for electromagnetic waves in optical helical waveguides (Nicolet et al., 2004; Nicolet and Zola, 2007) and for twisted electrostatic problems (Nicolet et al., 2007).

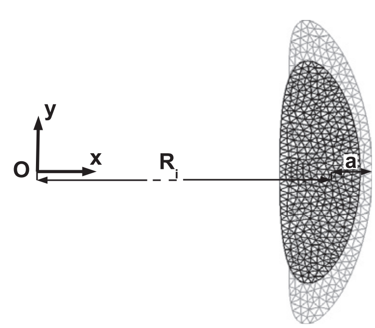

Fig. 1. Cross-section FE mesh of a helical waveguide with $R_i/a = 10$ and $\Phi_i = 75^\circ$. Grey: initial mesh ($E^E = 0$), black: updated mesh ($E^E = 40\%$), plotted in the initial and updated twisting coordinate system respectively.