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Hyperbolic heat conduction and associated transient thermal fracture for a piezoelectric material layer

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ABSTRACT

This paper studies the fracture mechanics of a piezoelectric material layer with an internal crack under the framework of hyperbolic, non-Fourier heat conduction. The paper includes two parts. The first part is for the case of a heated crack, in which the internal crack can be a source of heating (or cooling). This case develops the mode I thermal stress intensity factor at the crack tip. The second part is for the thermally insulated crack, which does not allow any penetration of the thermal flow across the crack. This case develops the mode II thermal stress intensity factor at the crack tip. Numerical results for the thermal stress intensity factor are plotted to show the effects of the thermal relaxation time, the crack length and the layer thickness. Comparisons between the non-Fourier results and the classical Fourier results are made. Limiting cases of the current problem include the solutions of thermoelastic crack problem and fracture mechanics associated with classical Fourier heat conduction.

1. Introduction

The classical Fourier heat conduction law assumes that the speed of heat propagation is infinite. This means that a thermal disturbance in a material can be felt instantaneously anywhere in the material. The accuracy of Fourier's heat conduction law is sufficient for many practical engineering applications. However, this theory cannot accurately explain conduction of heat caused by highly-varying thermal loading such as pulsed laser heating. For example, as pointed out by Babaei and Chen (2008), the surface temperature of a slab measured immediately after a sudden thermal shock is 300 °C higher than that predicted by Fourier's law (Maurer and Thompson, 1973). The Fourier heat conduction theory also breaks down at very low temperatures and when the applied heat flux is extremely large. Recent experimental and numerical studies have also reported breakdown of Fourier's law in nanomaterials, even if the phonon mean free path is much shorter than the characteristic length (Tzou, 1995a). The mechanism may lie in the ultrahigh-rate heat flux resulted from the extremely high temperature gradient or the extremely small cross sectional area. In these cases, the traditional continuum assumption may be still valid, even if the characteristic length falls into nanoscale (Wang et al., 2011). Such phenomena are of great interests because of much more potential technical and engineering applications, but still lack fundamental understandings. To better explain heat conduction in solids, non-Fourier heat conduction theories have been developed.

One of the non-Fourier theories is hyperbolic heat conduction theory. Cattaneo (1958) and Vernotte (1958) first independently introduced an additional material parameter, the thermal relaxation time, to generate a modification of Fourier's Law. The thermal relaxation time introduced in the theory is the time that the temperature field needs to adjust itself to thermal disturbances. This theory is called hyperbolic heat conduction because it results in a hyperbolic differential equation for temperature rather than the parabolic one obtained using Fourier's law. Since the proposal of the hyperbolic heat conduction model, several solutions have been given in the literature. Gembarovic and Majernik (1988) investigated non-Fourier effects in an insulated finite slab with a surface heat flux boundary conditions using Laplace transform. Lewandowska and Malinowski (1998) solved the hyperbolic equation for a semi-infinite body with a heat source. Later, Lewandowska and Malinowski (2006) present an analytical solution for the case of a thin slab symmetrically heated on both sides. Abdel-Hamid (1999) modeled the non-Fourier heat conduction in a finite medium subjected to a periodic heat flux using the finite integral transform technique. Also considered are the non-Fourier heat conductions in a finite medium for the case of an arbitrary periodic (Moosaie, 2007) and non-periodic (Moosaie, 2008) surface disturbances. Recently, Chen (2010) studied the hyperbolic heat conduction problems in cylinders using a hybrid Green's function method. Atefi and Talaee (2011) established an analytical solution for the non-Fourier axisymmetric temperature field within a finite hollow cylinder by using the hyperbolic heat conduction. Torabi and

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Saedodin (2011) analytically and numerically investigated the hyperbolic heat conduction in cylindrical coordinates subjected to heat flux boundary conditions. Solutions for the non-Fourier hyperbolic heat conduction in a functionally graded heterogeneous sphere (Babaei and Chen, 2008) and hollow cylinder (Babaei and Chen, 2010) were also presented. In addition, Keles and Conker (2011) conducted the non-Fourier hyperbolic heat conduction analysis for heterogeneous hollow cylinders and spheres made of functionally graded material. Clearly, most studies on the non-Fourier heat conduction have been carried out for the development of solution methods for the hyperbolic heat conduction equation. A comprehensive review of numerical methods for such equation has been given by Miller and Haber (2008).

It is well known that components subjected to temperature variation usually give rise to defects or cracks, which will disturb the local thermal flow distribution. The high intensification of the temperature gradient may induce thermal stress that may cause rapid crack growth. Hence, development of analyzing methods that can effectively estimate the thermal flow distribution in the materials with cracks is essential. On the other hand, piezoelectric materials are usually operated at the high temperature or lower temperature environment. The temperature change in these materials can result in severe thermal stresses. Because of the brittle nature of piezoelectric materials, excessive thermal stresses can breakdown or reduce the functionality and reliability of these advanced materials. Therefore, evaluation of coupling thermo-electro-mechanical cracking of piezoelectric materials is a topic of great interest. Ueda and Ashida (2009) studied the problem of an infinite row of parallel cracks in a nonhomogeneous material strip under static mechanical and transient thermal loading conditions. Tsamasphyros and Song (2005) constructed a general solution to the mechanical and electric fields in a finite thermopiezoelectric plate containing an isolated crack. Qin and Mai (2002) established a boundary element formulation for the analysis of interaction between a hole and multiple cracks in piezoelectric materials. Gao and Wang (2001) presented an explicit treatment of the generalized 2D thermopiezoelectric problem of an interfacial crack between two dissimilar thermopiezoelectric media by means of the extend Stroh formalism. Qin (2000) obtained the General solutions for thermopiezoelectrics with various holes under thermal loading. Gao and Noda (2004) investigated the thermal-induced interfacial cracking of magnetoelectroelastic materials.

There have been some pioneering investigations for the thermal stresses around cracks in thermoelastic materials using the hyperbolic heat conduction model. Manson and Rosakis (1993a,b) proposed a solution to the hyperbolic heat conduction equation for a traveling point heat source around a propagating crack tip, and measured the temperature distribution at the tip of a dynamically propagating crack experimentally. Tzou (1990a) analyzed the thermal field around a moving crack tip and studied the effect of crack velocity on the properties of the thermal shock. Other research in the field include the analysis of thermal-shock waves induced by a moving crack-a heat-flux formulation (Tzou, 1990b), a suddenly-opening crack in a coupled thermoelastic solid with thermal relaxation (Brock and Hanson, 2006), the second sound in a cracked layer based on Lord–Shulman theory (Zamani et al., 2011). Recently, Chen and Hu (2012) gave a thermoelastic analysis of a cracked half-plane under a thermal shock based on the hyperbolic heat conduction theory. More recently, Chen and Hu (2012) studied the transient temperature and thermal stresses around a partially insulated crack in a thermoelastic strip under a temperature impact by using the hyperbolic heat conduction theory.

This paper establishes a solution technique for the thermal fracture of a piezoelectric material layer under the framework of hyperbolic heat conduction model. Laplace transform is used to solve the time-varying behavior of the thermoelectroelastic field. Transient crack tip stress intensity factor is obtained to show the influences of the non-Fourier effect, crack size and layer thickness. Solution methods for the problems of heated crack (Section 3) and thermally insulated crack (Section 4) are established separately.

2. Basic governing equations

In the X–Y–Z coordinate system, the constitutive equations for the thermal flux and balance equation for the temperature under hyperbolic heat conduction law are (Tzou, 1995a; Wang and Han, 2012):

$$q_i(X_i,t) = -k\frac{\partial T(X_i,t)}{\partial X_i} - \tau_q \frac{\partial q_i(X_i,t)}{\partial t}, \tag{1}$$

$$\rho c \tau_q \frac{\partial^2 T}{\partial t^2} + \rho c \frac{\partial T}{\partial t} = k\left(\frac{\partial^2 T}{\partial X^2} + \frac{\partial^2 T}{\partial Y^2} + \frac{\partial^2 T}{\partial Z^2}\right), \tag{2}$$

where q denotes the heat flux; k is the thermal conductivity coefficient, ρ is the mass density and c is the specific heat, and τ_q is the thermal relaxation time, which is related to the collision frequency of the molecules within the energy carrier. In has been assumed that the thermal and temperature fields are not affected by the mechanical fields and the heat source is neglected. The thermal conduction equation (2) must be solved for prescribed boundary and initial conditions. The initial conditions specify the temperature and thermal flux distributions at time zero. These are $T(X_j,0) = \overline{T}_0(X_j,0)$ and $q_i(X_j,0) = \bar{q}_{0i}(X_j,0)$. A quantity with an over bar means that the quantity is prescribed. From Eq. (1), the initial conditions for thermal flux distribution can also be understood as $\rho c \partial T(X_j,0)/\partial t = -\bar{q}_{0i,i}(X_j,0)$. In most cases, the initial temperature and thermal flux are zero. Consequently, the initial conditions are equivalent to $T(X_j,0) = 0$ and $\partial T(X_j,0)/\partial t = 0$.

Consider the plane problem of a piezoelectric layer so that all the field variables are functions of X and Z only. The poling direction of the piezoelectric layer is parallel to the positive Z-axis. Denote the displacements along the X- and Z-directions as u and w, respectively, and the electric potential as ϕ. Constitutive equations for piezoelectric materials polarized along Z-direction are

$$\sigma_{xx} = c_{11}\frac{\partial u}{\partial X} + c_{13}\frac{\partial w}{\partial Z} + e_{31}\frac{\partial \phi}{\partial Z} - \lambda_{11}T, \tag{3a}$$

$$\sigma_{zz} = c_{13}\frac{\partial u}{\partial X} + c_{33}\frac{\partial w}{\partial Z} + e_{33}\frac{\partial \phi}{\partial Z} - \lambda_{33}T, \tag{3b}$$

$$\sigma_{xz} = c_{44}\left(\frac{\partial u}{\partial Z} + \frac{\partial w}{\partial X}\right) + e_{15}\frac{\partial \phi}{\partial X}, \tag{3c}$$

$$D_x = e_{15}\left(\frac{\partial u}{\partial Z} + \frac{\partial w}{\partial X}\right) - \epsilon_{11}\frac{\partial \phi}{\partial X}, \tag{3d}$$

$$D_z = e_{31}\frac{\partial u}{\partial X} + e_{33}\frac{\partial w}{\partial Z} - \epsilon_{33}\frac{\partial \phi}{\partial Z} - g_{33}T, \tag{3e}$$

where σ_{ij} and D_i $(i,j = x,z)$ are stresses and electrical displacements; c_{ij}, e_{ij} and ϵ_{ii} are elastic constants, piezoelectric constants and dielectric permittivities, respectively, λ_{ii} are temperature-stress coefficients, g_{33} is temperature-electric displacement coefficient. In the absence of body forces and body charges, the equilibrium equations are given by

$$\frac{\partial \sigma_{xx}}{\partial X} + \frac{\partial \sigma_{xz}}{\partial Z} = 0, \quad \frac{\partial \sigma_{xz}}{\partial X} + \frac{\partial \sigma_{zz}}{\partial Z} = 0, \quad \frac{\partial D_x}{\partial X} + \frac{\partial D_z}{\partial Z} = 0, \tag{4}$$

which can be expressed in terms of displacements and electric potential with the substitution of constitutive equations (3). Note that the electroelastic responses in the thermal stress approach here are assumed to be instant. The approach used here has already been adopted by other researchers for the determination of thermoelastic deformation associated with non-Fourier heat conduction (Brock and Hanson, 2006). It provides an easy yet sufficiently accurate