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Electromechanical and dynamic analyses of tunable dielectric elastomer resonator

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ABSTRACT

When used as resonators, dielectric elastomers are subjected to high frequencies and nonlinear oscillation. The present study is focused on a dielectric elastomer resonator whose dielectric membrane is subject to combined loads of tensile forces and voltages. When the loads are static, the resonator may reach a state of equilibrium. The stability and the natural frequency of the resonator with small-amplitude oscillation around the equilibrium state are analyzed. When a periodic voltage is applied, the device resonates at multiple frequencies of excitation. Pre-stretches and applied static voltages tune the natural frequency and modify the dynamic behavior of the resonator. The membrane may suffer loss of tension and electromechanical instability, causing the failure of the resonator. Safe operation range is identified for failure prevention while actuating the resonator.

1. Introduction

Dielectric elastomer (DE) consists of a soft elastomeric membrane sandwiched within two compliant electrodes on both sides. When a voltage is applied through the thickness, the DE membrane contracts in thickness and expands in area. Transducers made of DE possess excellent deformability, flexibility, affordability, and chemical and biological compatibility (Pelrine et al., 2000; Wissler and Mazza, 2005a,b; Carpi et al., 2008, 2010). Recently, DE transducers have been widely used as resonators, artificial muscles, adaptive optical elements, and programmable haptic surfaces (Bar-Cohen, 2002; Xia et al., 2005; Anderson et al., 2010; Mckay et al., 2010).

Most of the previous studies on DEs were focused on quasi-static deformation, with the effect of inertia neglected. In recent applications such as resonators, DEs can operate at a frequency as high as 50 kHz (Bonwit et al., 2006) in many applications and function as vibration sources. DE resonators exhibit promising advantages over conventional resonators since their natural frequencies can be set by structural parameters during fabrication and can be actively tuned by the applied static voltage. Researchers have designed DE resonators with different configurations. In a large-strain, high-frequency application, the elastomer membrane undergoes nonlinear oscillation. The dynamic behavior of a DE membrane resonator under voltages and pressures has been investigated (Mockensturm and Goulbourne, 2006; Fox, 2007; Fox and Goulbourne, 2008, 2009). Dubois et al. (2008) demonstrated the active tuning of resonant frequencies by applying voltage on a circular membrane resonator. Zhu et al. (2010a) and Yong et al. (2011) theoretically studied the dynamic behavior of DE membrane resonators. Feng et al. (2011) studied the oscillation of a DE-based micro-beam resonator and predicted the performance of the device against the Q-factor and the resonant frequency shift ratio. Interesting phenomena under electromechanical loads have been reported in DE actuators. Kollosche et al. (2012) investigated the 'pure-shear' DE actuators and observed that the voltage-deformation transition, the electromechanical instability and the loss of tension may occur during actuation, while the pre-stretches significantly alter the behavior of actuators. Keplinger et al. (2011) and Li et al. (submitted for publication) investigated a DE-based membrane inflation actuator and showed that the voltage-deformation response of the actuator was dictated by the mechanical loading paths. Explored also are the modes of failure that limit the performance of DE transducers (Wissler and Mazza, 2005a,b; Plante and Dubowsky, 2006, 2007; Carpi et al., 2010). Artificial Muscle Inc. (AMI) has fabricated a resonator recently, named Reflex™ HIC Slide Actuator HIC-512 (Biggs and Hitchcock, 2010). The resonator consists of soft dielectric membranes, a rigid frame and mass bars. The dynamic performances, such as strokes and acceleration responses, of the resonator with different input frequencies were examined in experiments. However, the pre-stretch effect and loading paths effect on actuation, loss of tension, electromechanical instability, and active tuning of the resonate frequency have yet to be studied for this DE-based resonator.

The aim of this paper is to develop an analytical model for the tunable dielectric elastomer resonator, with specific configuration referred to the Reflex™ HIC Slide Actuator HIC-512. The attention

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is focused on the pre-stretch and loading paths effects on actuation, as well as loss of tension, electromechanical instability, and active tuning of the resonate frequency. The work is presented as follows. Section 2 derives the equation of motion using the method of virtual work. Section 3 describes the state of equilibrium under static loads. Section 4 studies the small oscillation around the equilibrium state, as well as the tuning of the natural frequency of the resonator by pre-stretches and static applied voltages. Section 5 studies the parametric excitation with sinusoidal voltage.

2. Equation of motion

When an AC voltage is applied, the resonator oscillates around the state of equilibrium. The small-amplitude oscillation is governed by the equation of motion. To analyze the electromechanical behavior of the DE resonator, we derive the equation of motion under oscillation considering the inertia effect. Fig. 1 shows the schematics of the resonator. Fig. 1a displays a DE membrane of thickness H, with length L_1 in the 1-direction and L_2 in the 2-direction in a non-deformed state. The membrane is pre-stretched with the stretch λ_{1p} in the 1-direction and λ_{2p} in the 2-direction, as illustrated in Fig. 1b. The membrane is then attached to a rigid frame to maintain the pre-stretch. Two rigid mass bars with lumped mass m sandwich the dielectric elastomer membrane and divide it into two parts, marked as membrane A and membrane B. As shown in Fig. 1a, L_{1A} and L_{1B} are the original lengths along the 1-direction of membranes A and B in the undeformed state, with $L_{1A} + L_{1B} = L_1$. After the pre-stretch, membrane A is deformed to $\lambda_{1p}L_{1A}$ along the 1-direction and is coated on both surfaces with compliant electrodes as the active part. The passive part, membrane B, is deformed to $\lambda_{1p}L_{1B}$ in the 1-direction. In the actuated state, a voltage Φ_A is applied to the two electrodes of membrane A, as depicted in Fig. 1c. Membranes A and B have the deformed lengths $L_{1A}\lambda_{1A}$ and $L_{1B}\lambda_{1B}$ in the 1-direction, and $L_2\lambda_{2A}$ and $L_2\lambda_{2B}$ in the 2-direction. Here λ_{1A}, λ_{1B}, λ_{2A}, and λ_{2B} denote the stretches. Because the DE membranes are incompressible, membranes A and B have the deformed thicknesses $H/(\lambda_{1A}\lambda_{2A})$ and $H/(\lambda_{1B}\lambda_{2B})$. Electrons flow from one electrode of membrane A to the other and the two electrodes gain charges $+Q_A$ and $-Q_A$. Viewing membrane A as a parallel capacitor, one may calculate the charge on the electrode as

$$Q_A = \Phi_A \frac{\varepsilon L_{1A} L_2}{H} \lambda_{1A}^2 \lambda_{2A}^2, \tag{1}$$

where ε is the permittivity of the elastomer. Following the deformation of membranes A and B, mass bars move to a new position. During actuation, the membranes deform mainly in plane. The total length of the membranes in the 1-direction would remain constant until loss of tension happens in the 1-direction and membranes wrinkle, so that $L_{1A}\lambda_{1A} + L_{1B}\lambda_{1B} = L_1\lambda_{1p}$.

In the spirit of the recent theoretical studies of elastic dielectrics (Dorfmann and Ogden, 2005; McMeeking and Landis, 2005; Suo et al., 2008), one may state the thermodynamics of elastic dielectrics and the kinematics of membranes A and B. Membrane A represents a thermodynamic system of three independent variables λ_{1A}, λ_{2A} and Φ_A, while membrane B of λ_{1B}, λ_{2B} and Φ_B. Both membranes A and B are held at the ambient temperature. Membranes A and B are characterized by the Helmholtz free energy densities $W_A(\lambda_{1A}, \lambda_{2A}, \Phi_A)$ and $W_B(\lambda_{1B}, \lambda_{2B}, \Phi_B)$, which represent their total free energies in the deformed state scaled by the volume of the membranes in an undeformed state. Both systems of membranes A and B evolve with time t. As shown in Fig. 2, membrane A is subject to the voltage Φ_A and tensile forces P_{1A} and P_{2A} in the 1- and 2-directions, while membrane B is only subject to tensile forces P_{1B} and P_{2B}.

When the kinematic variables, λ_{1A} and λ_{2A}, vary slightly, the free-energy density of membrane A changes accordingly, and the tensile forces do work of $P_{1A}L_{1A}\delta\lambda_{1A}$ and $P_{2A}L_2\delta\lambda_{2A}$. When the charge on the electrode varies by δQ_A, the applied voltage does a work of $\Phi_A\delta Q_A$.

During actuation, the inertia force in each material element along the 1-direction is: $-\rho L_2 H x^2(\mathrm{d}^2\lambda_{1A}/\mathrm{d}t^2)$. The total work done by the initial force can be integrated along the length direction: $-\rho L_2 H(\mathrm{d}^2\lambda_{1A}/\mathrm{d}t^2)\delta\lambda_{1A}\int_0^{L_{1A}} x^2\mathrm{d}x$, which gives $(-L_{1A}^3\rho L_2 H/3)(\mathrm{d}^2\lambda_{1A}/\mathrm{d}t^2)\delta\lambda_{1A}$, where ρ denotes the density of the elastomer membrane. The same result can be obtained by the variation of kinetic energy:

$$\begin{aligned} -\delta\left(\int_0^{L_{1A}} \tfrac{1}{2}\rho L_2 H x^2 \left(\tfrac{\mathrm{d}\lambda_{1A}}{\mathrm{d}t}\right)^2 \mathrm{d}x\right) &= -\tfrac{\partial}{\partial t}\left(\int_0^{L_{1A}} \tfrac{1}{2}\rho L_2 H x^2 \left(\tfrac{\mathrm{d}\lambda_{1A}}{\mathrm{d}t}\right)^2 \mathrm{d}x\right)\delta t \\ &= -\rho L_2 H \tfrac{\mathrm{d}^2\lambda_{1A}}{\mathrm{d}t^2}\tfrac{\mathrm{d}\lambda_{1A}}{\mathrm{d}t}\delta t \int_0^{L_{1A}} x^2 \mathrm{d}x \\ &= -\tfrac{L_{1A}^3}{3}\rho L_2 H \tfrac{\mathrm{d}^2\lambda_{1A}}{\mathrm{d}t^2}\delta\lambda_{1A}. \end{aligned} \tag{2}$$

For an arbitrary variation of the system, the variation in the free energy of membrane A is equal to the work done jointly by the voltage, the tensile forces and the inertia force,

$$\begin{aligned} L_{1A}L_2 H\delta W_A = \Phi_A\delta Q_A + P_{1A}L_{1A}\delta\lambda_{1A} + P_{2A}L_2\delta\lambda_{2A} - \frac{L_{1A}^3}{3}\rho L_2 H \\ \times \frac{\mathrm{d}^2\lambda_{1A}}{\mathrm{d}t^2}\delta\lambda_{1A}. \end{aligned} \tag{3a}$$

Similarly, for membrane B, with $\Phi_B = 0$, $Q_B = 0$, one has

$$L_{1B}L_2 H\delta W_B = P_{1B}L_{1B}\delta\lambda_{1B} + P_{2B}L_2\delta\lambda_{2B} - \frac{L_{1B}^3}{3}\rho L_2 H\frac{\mathrm{d}^2\lambda_{1B}}{\mathrm{d}t^2}\delta\lambda_{1B} \tag{3b}$$

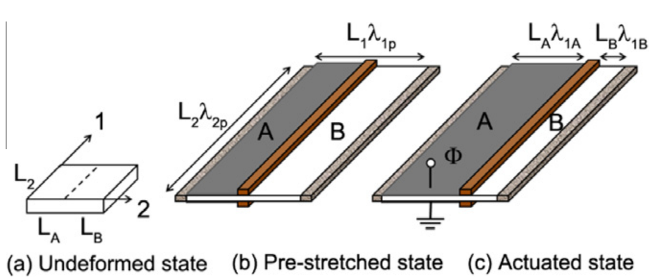

Fig. 1. Schematics of a DE resonator in three states. (a) Undeformed state. The membrane is of length L_1 in the 1-direction, length L_2 in the 2-direction, and thickness H. (b) Pre-stretched state. The membrane is pre-stretched in the 1- and 2-directions and is held by rigid frames. The membrane is of length $L_1\lambda_{1p}$ in the 1-direction and length $L_2\lambda_{2p}$ in the 2-direction. Rigid bars with lumped mass m sandwich the membrane and divide it into membrane A and membrane B. Electrodes are coated on both sides of membrane A. (c) Actuated state. In response to the voltage Φ_A applied on membrane A, membrane A is of length $L_A\lambda_{1A}$ in the 1-direction while membrane B is of length $L_B\lambda_{1B}$ in the 1-direction, and the mass bar moves accordingly.

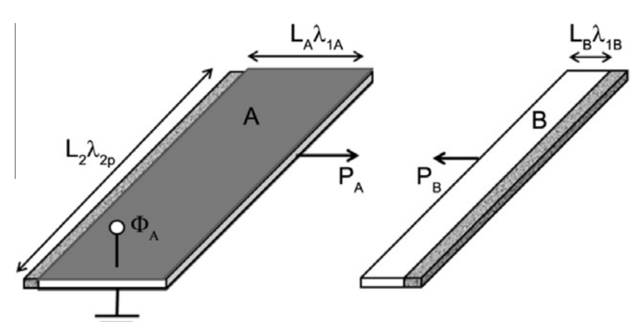

Fig. 2. Schematics of membranes A and B in an actuated state. Membrane A is subject to a voltage Φ_A and a tensile force P_{1A} in the 1-directoin. Membrane B is subjected to a tensile force P_{1B} in the 1-directoin. P_{1A} is equal to P_{1B}