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Statistical size effect of flexural members in steel structures

Zheng Li *, Hartmut Pasternak

Chair of Steel and Timber Structures, Brandenburg University of Technology, Cottbus, Germany

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ABSTRACT

This paper presents the results of theoretical and experimental investigations on the statistical size effect (SSE) of flexural members in steel. The chain of bundle model is developed, which is based on a stochastic material model by Weibull and lognormal distribution. Furthermore, this model is embedded into the finite element method (FEM) software for the analysis of a complicated structure with a stress gradient. In order to determine the stochastic material model parameters, uniaxial tensile tests were carried out based on specimens using the same steel with different sizes. By comparing the experimental and simulation results, the material parameters could be analyzed based on the relationship between the specimen sizes and strength. Moreover, the 3-point and 4-point bending tests were performed and simulated with the developed model. The experimental and simulation results demonstrate that the SSE also exists in the flexural member, and the equivalent yield stress is closely related to the stress distribution and volume of the structure component.

1. Introduction

In the past several decades, the question has been raised by reliability analysis research, whether the material strength can be affected by the structure volume [1–4]. Previous research reported that the size effect can be described by two fundamentally different approaches, namely deterministic and statistical explanations. Most researchers have focused on the size effect on the energetic basis, and this purely deterministic size effect has been extensively studied [5,6]. Traditionally, the statistical size effect has been explained by Weibull-type statistical weakest link model [7], where the material strength is dependent on the weakest member. The basic hypothesis is that the structure will fail as the stress exceeds the material strength at any point. Weibull distribution, which is based on the weakest link model, has been widely applied to brittle material owing to its simple form and relative preciseness [8,9]. However, recent research has indicated that the SSE based on Weibull distribution for rock, concrete and other quasi-brittle material should be corrected [10].

The SSE of steel structures was mentioned decades ago [2]. However, currently researches in this area are rare. This is due to the fact that the SSE in steel structures is not as prominent as in concrete since the strength variability of steel is relatively small. Moreover, most of the components in the steel structure are plate-type and not bulk-type, and some studies have focused on the relationship between strength and material thickness. For example, Fig. 1a) shows that the material yield strength decreases with increasing material thickness. Thus, the material strength is graded by thickness in Fig. 1b), but this method ignores the influence of the tensile specimen size. Hence, the graded material strength according to the thickness strength in the reference [11] and design code [12] covers the SSE phenomenon. Theoretically, all materials are imperfect and consist of defective structures on a microscopic scale. The distribution of defects on the microscale determines the material strength on the macroscale. Because of the randomly distributed sliding surfaces and other mechanical defects in materials, it is possible to study the SSE by means of statistical methods. Recently, the SSE in a steel structure was demonstrated by experiments [13]. Moreover, the results of [14] indicated that the influence of the SSE on the reliability of steel structures cannot be ignored.

Generally, flexural strength is determined based on tensile strength, according to mechanical methods using the plastic theory. Because steel is regarded as an ideal body, the microscopic structure imperfections and real stress distributions in the structure component cannot be considered. Theoretically, the stochastic finite element method with correlated random fields can be used to analyze the SSE in steel caused by the material's microstructure imperfection [14]. However, the estimation of the random field parameters, e.g. coefficient of variation and correlation length, and the efficiency of this approach for nonlinear materials limit the large-scale application of the stochastic finite element method. The stochastic material model provides a possible means of estimating the influence of the stress gradient and stressed volume on strength. This paper aims to describe the SSE using a proposed stochastic material model that is based on the existing material models as well as probability theory. Furthermore, the model is programmed using User Subroutines in ABAQUS to describe the SSE in flexural members and complex structures.

* Corresponding author.
E-mail address: Zheng.Li@b-tu.de (Z. Li).

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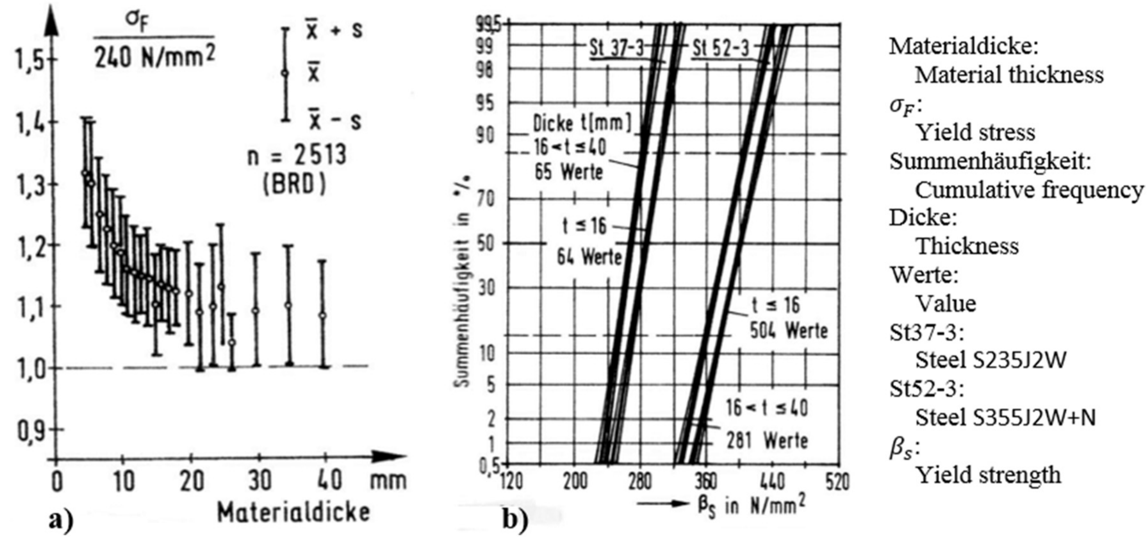

Fig. 1. a) Relation between yield stress and material thickness, b) the probability distribution of the strength of the steel graded according to the thickness, from [11].

2. Stochastic material models

Theoretically, artificially manufactured materials such as steel are not completely isotropic and homogeneous, and contain various microstructure imperfections. Currently, the prediction of failure of structural elements based on microstructural analysis, such as Gurson-Tvergaard-Needleman (GTN) material model [15], is widely mentioned and studied. However, these researches did not deal with the link between the randomness of the underlying microstructure imperfections and the probability distribution of material properties at macroscopic scale. The material properties are affected by the structural size, since the size, quantity and corresponding distribution of microstructure imperfections are changed as the structural volume increases. From a statistical point of view, increased defects can lead to a greater probability of failure under the same stress conditions; that is the strength decreases as the failure probability remains unchanged. According to certain simplified assumptions of the strength on the microscale, the macroscopic material strength can be obtained by using a stochastic material model with probability and statistical methods. This method, based on the strength of the statistical theory, avoids the difficulty of researching the material microstructure.

Steel is not an ideal elasto-plastic material and the post-peak behaviors of steel after the elastic limit is harder to analyze as the dislocations in the crystalline structure start moving after the yield point [16]. Therefore, the two classical models based on limiting states for probabilities [17], namely the weakest link model proposed by Weibull [7] and the fiber bundle model by Daniels [18], cannot describe real material properties clearly and precisely [19]. As an alternative approach [20], real materials can be described using the chain of bundle model [21,22], which is based on both the classical models.

The chain of bundle model, depicted in Fig. 2, is composed of N representative volume elements (RVEs) of n parallel reference elements in a chain. An RVE is defined as the smallest material element, whose failure can cause destruction of the entire structure. If the RVE can result in total failure, the structure can be simplified as an RVE chain. It is assumed that the element number of the parallel system is infinite; the statistics of the respective parallel system can be analyzed and follow a normal distribution. For a large coefficient of variation, the normal distribution should be replaced by a logarithmic normal distribution to reduce the heterogeneity variance of the material property and prevent negative strength. It is assume that the failure probabilities of the RVE are statistically uncorrelated. For the chain of bundle model, the probability of failure, or cumulative distribution function, is expressed as follows:

$$F_p(\sigma) = 1 - e^{\frac{V}{V_{RVE}} * \ln\left(1 - \varnothing\left(\frac{\ln\left(\frac{\sigma - \sigma_u}{\sigma_0}\right)}{\xi}\right)\right)} \approx 1 - e^{-\frac{V}{V_{RVE}} * \varnothing\left(\frac{\ln\left(\frac{\sigma - \sigma_u}{\sigma_0}\right)}{\xi}\right)} \quad (1)$$

where $\varnothing(*)$ is the distribution function of the standard normal distribution; ξ is the material constant; σ_u is the lower limit of σ and $\sigma > \sigma_u$; σ_0 is a scale parameter of strength.

Eq. (1) provides a continuous transition between the Weibull weakest link model and Daniel's parallel bundle models; it is purely phenomenological and not based on any physical models. The failure probability density function approximates to a lognormal distribution if the entire specimen volume is extremely small; that is $V/V_{RVE} \rightarrow 1$.

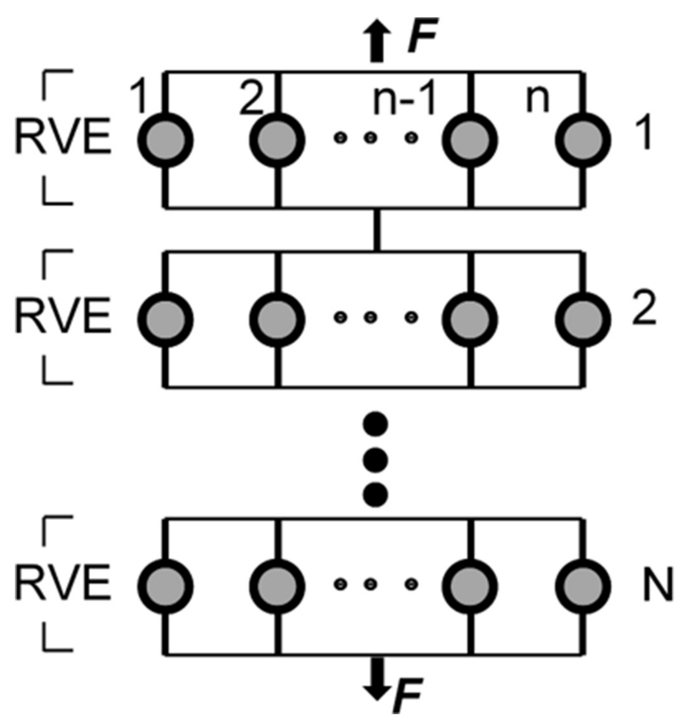

Fig. 2. Chain of bundle model.