Journal of Constructional Steel Research 141 (2018) 226–240

Contents lists available at ScienceDirect

Journal of Constructional Steel Research

Global stability analysis of spatial structures based on Eigen-stiffness and structural Eigen-curve

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ARTICLE INFO

Article history:
Received 4 July 2017
Received in revised form 2 November 2017
Accepted 11 November 2017
Available online 5 December 2017

Keywords:
Spatial structure
Eigen-stiffness
Structural Eigen-curve
Global stiffness analysis

ABSTRACT

The global stability of a spatial structure is usually analysed via a nodal load-displacement curve (NLDC). However, the global mechanical properties of a structure can hardly be reflected comprehensively by specific NLDCs, and no criterion is available to select the representative node for getting the most reasonable NLDC. In this paper, a scalar parameter derived from the incremental equilibrium equation of nonlinear stability analysis, named Eigen-stiffness, is defined to characterize the global structural stiffness. The Structural Eigen-curve (SEC), based on Eigen-stiffness, is proposed to depict the equilibrium path. Two types of extreme points on SEC are defined to determine the critical state of the structure, including the structural limit state and the structural snap-back. Firstly, the stability of a hinge-supported planar arch is analysed to introduce the SEC concept. Then a K6 reticulated shell is designed to give further understanding. Subsequently, practical application of the SEC is illustrated in the stability analysis of a roof structure, namely, Shanghai International Conference Centre. In addition, a parametric study on a K6 reticulated shell is carried out, based on the Eigen-stiffness and the SEC, to investigate the effects of the rise to span ratio and the geometric imperfection amplitude on the structural stiffness and the structural load-carrying capacity. The results demonstrate that the ultimate load-carrying capacity obtained from the SEC is equal to that from NLDC. More importantly, unlike NLDC, the SEC — free from node selection — can efficiently capture the features of global structural behaviour and the evolution of structural stiffness.

1. Introduction

Ensuring static global stability is one of the most important issues in the structural design of spatial structures. Research on the stability of spatial structures has been conducted for decades, and plenty of results have been obtained, including nonlinear iteration methods [1–3], experimental tests [4–5], and numerical analysis results [6]. The stability of both typical-shaped structures (spherical shell [7], cylindrical shell [8], conical shell [9], etc.) and non-typical-shaped structures (inverted catenary cylindrical shell [10], barrelled shell [11], and egg-shaped shell [12], etc.) is still a hot topic and has been studied continually [13–16]. Generally, static global stability analyses include linear buckling analysis, geometrically and materially nonlinear analysis for both perfect and imperfect structures. For linear buckling analyses, the buckling factor of a given structure can be easily obtained via eigenvalue analysis. For nonlinear stability analyses, two main problems are involved: (i) how to capture the equilibrium point in nonlinear iteration; and (ii) how to explicitly depict the equilibrium path. For the former problem, several numerical methods have been proposed, including the most popular Newton-Raphson method (N-R method). However, the N-R method is limited in tracing the post-buckling path of a structure due to the singularity of the tangent stiffness matrix at the ultimate point. The arc-length method (A-L method) proposed by Riks [17] and modified by Crisfield [18] effectively solved the singularity-related problem in nonlinear iteration. To date, the A-L method is the most stable and efficient numerical method for tracing the full-range equilibrium path of a spatial structure in static stability analysis.

For problem (ii), the nodal load-displacement curve (NLDC) has been widely used. However, there is no valid criterion for selecting the representative nodes. The selection of representative nodes is usually based on the engineer's experience, commonly, the node with maximum displacement is chosen as the representative one [19]. Although the NLDC is widely used, several problems with NLDCs cannot be avoided: Is the node with maximum displacement always suitable for analysing the evolution of structural mechanical property? Since different NLDCs may give different features of a determined load-carrying process [20–21], which of these NLDCs is more reasonable for depicting the structural behaviour? Obviously, the NLDC just provides the ultimate load factor without other global information and only reflects the behaviour of local structures around the selected node, instead of the global structural behaviour. For a global point of view, not only the ultimate load factor should be concerned, but also the structural mechanical behaviour and some critical states in whole load-carrying

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https://doi.org/10.1016/j.jcsr.2017.11.003
0143-974X/

process, like structural stiffening and softening, as well as dramatically changing in structural stiffness, etc. Due to the direct influence of structural stiffness on the structural stability, many studies focused on the structural stiffness [22–27]. For reticulated shell, based on the assumption of equivalent continuum [22], the global structural stiffness was proposed as equivalent stiffness [22–23] which would be computed from the equivalent bending stiffness and equivalent membrane thickness [28]. As equivalent stiffness is determined by the initial state of a structure, it does not change during the load-carrying process. For this reason, current stiffness parameter (CSP) was suggested to characterize the global behaviour in the solution process [24]. According to CSP, it can be inferred from the value of CSP that whether the structure is softening or stiffening [29]. As the value of CSP tends to zero, the structure becomes unstable. However, the CSP is a dimensionless quantity that cannot reflect the influence of various structural parameters, such as geometric parameters, on structural stiffness. Therefore, this quantity is generally used to determine travel directions in path following [30–31]. Based on the concept of CSP, Xiang et al. derived an energy-based stiffness parameter to describe the structural stiffness of latticed arches [25]. In their further studies, this parameter was extended to incorporate structural vibrating properties, and was used for establishing the simplified dynamic models of complex cable net curtain walls [26] and steel roofs [27]. But the parameter proposed by Xiang et al. is established using vibrating-mode-proportioned loads, which is certainly not suitable for structural stability analysis. As a matter of fact, the structural stiffness is related to a number of structural parameters (rise to span ratio, amplitude of imperfection, etc.), thus the structural sensitivity to these parameters is also an important aspect in stability analysis [20,32]. Unfortunately, the effect of these parameters are just reflected by ultimate load factor and their effects on structural mechanical properties (structural stiffness, structural behaviour, etc.) are not addressed.

The present study aims to find a stiffness-related parameter that can characterize the global mechanical behaviour of structures and quantify the effect of various structural parameters in stability analysis. In doing so, a new scalar parameter defined as Eigen-stiffness is derived from the equilibrium equation in the present study, providing a quantitative and continuous measure for describing global structural stiffness. Subsequently, the Structural Eigen-curve (SEC) is proposed and its properties are illustrated in detail. Two numerical models are designed to show the advantages of the SEC and a parametric study on the K6 shell was conducted. Moreover, stability analysis for a practical engineering application was carried out using the Eigen-stiffness and SEC. The final section summarizes the results and conclusions.

2. Basic theory for Eigen-stiffness

2.1. Definition of Eigen-stiffness

In nonlinear structural analysis, the relationship between the incremental displacement and the incremental load is generally established by the incremental equilibrium equation as follows:

$$\boldsymbol{K}_{\mathrm{T}}\Delta\boldsymbol{U} = \Delta\boldsymbol{P} \tag{1}$$

where $\boldsymbol{K}_{\mathrm{T}}$ is tangent stiffness matrix, $\Delta\boldsymbol{U}$ is incremental displacement vector, $\Delta\boldsymbol{P}$ is incremental load vector. For a generic iteration step i, the incremental equilibrium equation can be written as

$$\boldsymbol{K}_{Ti}\Delta\boldsymbol{U}_i = \Delta\boldsymbol{P}_i \tag{2}$$

As the applied load is a conservative force, the incremental load can be expressed as

$$\Delta\boldsymbol{P}_i = \Delta\chi_i\boldsymbol{\varphi} \tag{3}$$

where $\Delta\chi_i$ is the incremental load factor in the i-th iteration step, $\boldsymbol{\varphi}$ is the load pattern mode.

Multiply each side of Eq. (2) by the transpose of $\Delta\boldsymbol{U}_i$

$$\Delta\boldsymbol{U}_i^{\mathrm{T}}\boldsymbol{K}_{\mathrm{T}i}\Delta\boldsymbol{U}_i = \Delta\boldsymbol{U}_i^{\mathrm{T}}\Delta\boldsymbol{P}_i \tag{4}$$

transform the left side of Eq. (4) as

$$\frac{\Delta\boldsymbol{U}_i^{\mathrm{T}}\boldsymbol{K}_{\mathrm{T}i}\Delta\boldsymbol{U}_i}{\Delta\boldsymbol{U}_i^{\mathrm{T}}\Delta\boldsymbol{U}_i}\Delta\boldsymbol{U}_i^{\mathrm{T}}\Delta\boldsymbol{U}_i = \Delta\boldsymbol{U}_i^{\mathrm{T}}\Delta\boldsymbol{P}_i \tag{5}$$

and then define the Eigen-stiffness as k^*

$$k_i^* = \frac{\Delta\boldsymbol{U}_i^{\mathrm{T}}\boldsymbol{K}_{\mathrm{T}i}\Delta\boldsymbol{U}_i}{\Delta\boldsymbol{U}_i^{\mathrm{T}}\Delta\boldsymbol{U}_i} \tag{6}$$

$$\Delta u_i^* = \Delta\boldsymbol{U}_i^{\mathrm{T}}\Delta\boldsymbol{U}_i \tag{7}$$

$$\Delta p_i^* = \Delta\boldsymbol{U}_i^{\mathrm{T}}\Delta\boldsymbol{P}_i \tag{8}$$

where k_i^* is the Eigen-stiffness in accordance with iteration step i. By substituting Eqs. (6), (7), and (8) into Eq. (5), Eq. (5) can be transformed into the following form:

$$k_i^* \cdot \Delta u_i^* = \Delta p_i^* \quad , \qquad k_i^* = \Delta p_i^* / \Delta u_i^* \tag{9}$$

where Δu_i^* and Δp_i^* are the incremental Eigen-displacement and the incremental work in accordance with iteration step i, respectively. According to the transformation of Eq. (9), k^* is equal to the ratio of Δp^* to Δu^* in each iteration step, wherein Δp^* and Δu^* are calculated based on the load vector $\Delta\boldsymbol{P}$ and the displacement vector $\Delta\boldsymbol{U}$ (see Eqs. (7) and (8)). Since $\Delta\boldsymbol{P}$ and $\Delta\boldsymbol{U}$ can be directly outputted from FEA software, the Eigen-stiffness could be easily obtained by Eq. (9) via computer programming of MATLAB post-process.

According to Eq. (6), it can be found that k^* is related to the tangent stiffness matrix and does not rely on specific nodal displacement. The dimensions of k^* are the same as that of the conventional stiffness, and the value of k^* is numerically equal to the work done by incremental load $\Delta\boldsymbol{P}$ on the condition of $\Delta u^* = 1$. In the eigenspace of tangent stiffness matrix, when the angle between $\Delta\boldsymbol{P}$ and $\Delta\boldsymbol{U}$ is $<90°$, the work done by $\Delta\boldsymbol{P}$ is positive (Fig. 1a), and so is k^*; when the angle is $>90°$, both the work and k^* are negative (Fig. 1b). If the incremental load factor $\Delta\chi$ is equal to zero, $\Delta\boldsymbol{P}$ is actually transferred to the zero vector and k^* is equal to zero (Fig. 1c). In this case, the structure is in a critical state, better known as the limit state. However, it should be noted that the value of k^* is also equal to zero when $\Delta\boldsymbol{P}$ is perpendicular to $\Delta\boldsymbol{U}$ (Fig. 1d), in which another critical state occurs, called structural snap-back. Therefore, the condition of $k^* = 0$ is a necessary but insufficient condition to determine whether the structure is in the limit state or not.

2.2. Eigen-stiffness and tangent stiffness matrix

2.2.1. Spectral decomposition of K_T

Suppose that the eigenvalue of $\boldsymbol{K}_{\mathrm{T}}$ is λ_n and the corresponding eigenvector is $\boldsymbol{\phi}_n$. The characteristic equation of $\boldsymbol{K}_{\mathrm{T}}$ can be written as follows:

$$\boldsymbol{K}_{\mathrm{T}}\boldsymbol{\phi}_n = \lambda_n\boldsymbol{\phi}_n \quad (n = 1, 2, \ldots, N) \tag{10}$$

By sorting the eigenvalue λ_n $(n = 1,2, \ldots,N)$ in ascending order as $\lambda_1 \le \lambda_2 \le \ldots \le \lambda_N$, the tangent stiffness matrix $\boldsymbol{K}_{\mathrm{T}}$ can be converted as

$$\boldsymbol{K}_{\mathrm{T}} = \boldsymbol{\Phi}\boldsymbol{\Lambda}\boldsymbol{\Phi}^{-1} \tag{11}$$

$$\boldsymbol{\Lambda} = \begin{bmatrix} \lambda_1 & & & 0 \\ & \lambda_2 & & \\ & & \ddots & \\ 0 & & & \lambda_N \end{bmatrix} \tag{12}$$