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Lateral–torsional buckling of steel web tapered tee-section cantilevers

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ABSTRACT

In this paper an analytical model is presented to describe the lateral–torsional buckling behaviour of steel web tapered tee-section cantilevers when subjected to a uniformly distributed load and/or a concentrated load at the free end. To validate the present analytical solutions finite element analyses using ANSYS software are also presented. Good agreement between the analytical and numerical solutions is demonstrated. Using the present analytical solutions, the interactive buckling of the tip point and uniformly distributed loads is investigated and a parametric study is carried out to examine the influence of section dimensions on the critical buckling loads. It is found that web tapering can increase or decrease the critical lateral–torsional buckling loads, depending on the flange width of the beam. For a beam with a wide flange (width/depth = 0.96) the critical buckling load is increased by 2% by web tapering, whereas for a beam with a narrow flange (width/depth = 0.19) web tapering reduces the buckling load up to10% and 6% for the tip point loading and the uniformly distributed load respectively.

1. Introduction

Tee-section beams are widely used in modern construction due to their structural efficiency. The main feature of a tee-section beam is the monosymmetry of the cross-section. For most tee-section cantilever beams carrying gravity loading the flange is positioned at the top, in which case the flange is in tension and the unstiffened portion of the web is in compression. Because the neutral axis of a tee-section beam is closer to the flange the maximum compressive stress in the web is much higher than the maximum tensile stress in the flange. This means that such beams fail by compressive stress and the lateral–torsional or lateral–distortional buckling could be one of the main failure modes [1].

The instability of monosymmetric I-beams under various loading conditions has been studied by many researchers [2–8]. The main difficulty of the problem is the presence of an additional torque, owing to the monosymmetry of the section, arising from the pre-buckling longitudinal bending stresses as the beam twists during the buckling. This additional torque causes an effective change in the torsional stiffness of the beam. This feature does not exist in symmetric beams, and was not addressed until the 1940s [9]. Since then different modifications of the torsional stiffness to account for the effect of the additional torque have been proposed [6].

Steel web tapered tee-section beams are very popular because of their aesthetic features and light weight. These beams are mainly cantilevered and have the advantage of low weight-to-strength ratios. They are structurally efficient since the web can be tapered along the beam to closely match the variation of the bending moment of the beam. The depth of the beam is largest at the fixed support, where its bending moment is greatest, and gradually decreases towards the free end.

Although steel web tapered tee-section cantilevers are commonly used, research into the instability of such beams is very limited. The majority of the existing literature deals with the lateral–torsional buckling of tapered I-beams [10–18]. Studies into tapered tee-section cantilevers are few. One rare example is by Fischer and Smida [19]. Kitipornchai and Trahair [10] derived differential equations for the non-uniform torsion of tapered I-beams by analyzing the deformations of the flanges and investigated the elastic flexural–torsional buckling of simply supported tapered I-beams. Later, they extended their method to tapered mono-symmetric I-beams. [11]. Yang and Yau [13], and Bradford and Cuk [14] presented numerical investigations on the lateral–torsional and lateral–distortional buckling of tapered monosymmetric I-beams using finite element methods.

Studies by Andrade et al. [16,17] have shown that the lateral torsional buckling loads of simply supported web tapered I-beams were decreased by as much as 20 to 40% as the degree of taper increased. This disagrees with the earlier work by Kitipornchai and Trahair [10], which concluded that the critical loads of web tapered beams did not vary greatly as the degree of taper increased since the torsional stiffness was insensitive to the degree of taper. The disagreement could be because a very short beam of 1.52 m span was used in the work of Kitipornchai and Trahair [10], while Andrade et al. [16,17] used long beams of 6 m, 9 m, and 12 m.

The boundary conditions of a beam seem to influence the lateral torsional buckling loads of web tapered I-beams. The buckling loads are decreased for simply supported beams [16–18], while being increased for fix-end beams [18] and cantilevers [16–18], compared with those of un-tapered ones. The increase for the cantilevers is significant and increases as the degree of taper increases. However those three studies

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[16–18] are based on analytical and numerical analyses and hence the results should be validated by experimental work.

Web tapered tee-section cantilevers may behave differently from tapered I-beam cantilevers. The lack of a bottom flange means that the lower part of the web is in compression and this increases buckling instability. A study by Fisher and Smida [19] discussed the failure modes of such beams, however their experimental results were not compared to any un-tapered beams. Furthermore no analytical study has been carried out on this topic.

In this paper an analytical model is presented to describe the lateral–torsional buckling behaviour of steel web tapered tee-section beams when subjected to a uniformly distributed load and/or a concentrated load at the free end. To validate the present analytical solutions finite element analyses using ANSYS software are also presented. Using the present analytical solutions, the interactive buckling of the distributed and concentrated loads is discussed and a parametric study is carried out to provide the optimum design of tee-section beams against lateral–torsional buckling.

2. Lateral–torsional buckling analysis of steel web tapered tee-section cantilevers

Consider a web tapered tee-section cantilever subject to a uniformly distributed load and a concentrated load at its free end, as shown in Fig. 1. Let x be the longitudinal axis of the beam, y and z be the cross-sectional axes parallel to the web and flange, respectively. For convenience, the origin of coordinates was chosen to be the centroid of the section. Due to the tapering of the web, the section properties of the beam are a function of the coordinate x and can be expressed as follows:

$$\bar{y} = \frac{\frac{b_f t_f^2}{2} + t_w(b_{wo} - x\tan\alpha)\left(t_f + \frac{b_{wo} - x\tan\alpha}{2}\right)}{b_f t_f + t_w(b_{wo} - x\tan\alpha)} \quad (1)$$

$$I_y = \frac{t_f b_f^3}{12} + \frac{(b_{wo} - x\tan\alpha)t_w^3}{12} \quad (2)$$

$$I_z = t_w(b_{wo} - x\tan\alpha)\left[\frac{(b_{wo} - x\tan\alpha)^2}{12} + \left(\frac{b_{wo} - x\tan\alpha}{2} + t_f - \bar{y}\right)^2\right] + b_f t_f\left[\frac{t_f^2}{12} + \left(\bar{y} - \frac{t_f}{2}\right)^2\right] \quad (3)$$

$$J = \frac{b_f t_f^3 + (b_{wo} - x\tan\alpha)t_w^3}{3} \quad (4)$$

where $\bar{y}$ is the distance from the top of the section to the neutral axis, I_y and I_z are the second moments of the cross-sectional area about the y- and z-axes, respectively, J is the torsional constant of the section, b_f is the flange width, t_f is the flange thickness, b_{wo} is the web depth at the support ($x = 0$), t_w is the web thickness, and α is the tapering angle.

Assume that when lateral–torsional buckling occurs, the displacements of the beam can be described as follows:

$$v(x) = \sum_{n=0} A_n\left(\frac{x}{l}\right)^{n+2} \quad (5)$$

$$w(x) = \sum_{n=0} B_n\left(\frac{x}{l}\right)^{n+2} \quad (6)$$

$$\phi(x) = \sum_{n=0} C_n\left(\frac{x}{l}\right)^{n+1} \quad (7)$$

where v and w are the transverse and lateral displacements of the beam defined at the shear centre, respectively, ϕ is the angle of rotation of the cross-section, A_n, B_n and C_n ($n = 0, 1, 2, \ldots$) are the constants to be determined, and l is the length of the beam. Note that the displacement functions assumed in Eqs. (5)–(7) satisfy the clamped boundary conditions ($v = w = \phi = 0$ and $dv/dx = dw/dx = 0$) at the support ($x = 0$).

The strain energy of a tee-section beam due to the buckling displacements can be calculated using the following formula [20]:

$$U = \int_o^l \left[\frac{EI_z}{2}\left(\frac{d^2v}{dx^2}\right)^2 + \frac{EI_y}{2}\left(\frac{d^2w}{dx^2}\right)^2 + \frac{GJ}{2}\left(\frac{d\phi}{dx}\right)^2\right]dx \quad (8)$$

where E is the Young's modulus and G is the shear modulus. Note that for a tee-section the warping constant is zero and thus no warping energy is involved in Eq. (8). Substituting Eqs. (5)–(7) into Eq. (8) yields

$$U = \int_o^l \frac{EI_z}{2l^4}\left[\sum_{n=0} A_n(n+1)(n+2)\left(\frac{x}{l}\right)^n\right]^2 dx + \int_o^l \frac{EI_y}{2l^4}\left[\sum_{n=0} B_n(n+1)(n+2)\left(\frac{x}{l}\right)^n\right]^2 dx + \int_o^l \frac{GJ}{2l^2}\left[\sum_{n=0} C_n(n+1)\left(\frac{x}{l}\right)^n\right]^2 dx. \quad (9)$$

The loss of the potential energy of the externally applied loads due to the buckling displacements can be calculated using the following formula [2,4,6,20]:

$$W = -\int_o^l M_z\phi\frac{d^2w}{dx^2}dx - \frac{1}{2}\int_o^l M_z\beta_z\left(\frac{d\phi}{dx}\right)^2 dx \quad (10)$$

where M_z is the internal bending moment about the z-axis of the beam in the pre-buckling stage, which is generated due to the externally applied loads, and β_z is the parameter describing the monosymmetric property

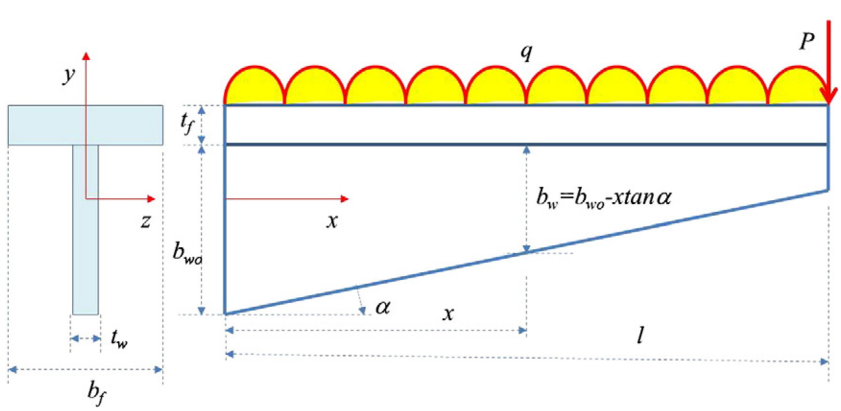

Fig. 1. A web tapered tee-section cantilever beam subject to a uniformly distributed load and a concentrated load at its free end.