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On the four-dimensional lattice spring model for geomechanics

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ABSTRACT

Recently, a four-dimensional lattice spring model (4D-LSM) was developed to overcome the Poisson's ratio limitation of the classical LSM by introducing the fourth-dimensional spatial interaction. In this work, some aspects of the 4D-LSM on solving problems in geomechanics are investigated, such as the ability to reproduce elastic properties of geomaterials, the capability of solving heterogeneous problems, the accuracy on modelling stress wave propagation, the ability to solve dynamic fracturing and the parallel computational efficiency. Our results indicate that the 4D-LSM is promising to deal with problems in geomechanics.

1. Introduction

Rock, concrete and soil are the main research subjects in geomechanics, which are commonly involved in various engineering activities, such as mining, tunnelling and underground engineering. Mechanical responses of these materials are the most concerns for researchers (Dai et al., 2010; Brown, 2015; Feng et al., 2016). For decades, the predecessors, e.g. Dr. K. Terzaghi, Dr. E. Hoek and Dr. E.T. Brown, have made fruitful works. However, some fundamental problems are still unclear. For example, an accurate constitutive model for deformation and a rational model (Liu and Carter, 2002) for three-dimensional (3D) crack propagation (Ingraffea, 1987; Yang et al., 2017) are still under exploring. These fundamental questions are directly related to major engineering safety issues, e.g. tunnel collapse, slope instability, rockburst, and nuclear waste leakage. Geomaterials are usually heterogeneous and their mechanical responses are highly nonlinear, which made them too difficult to be described through pure analytical study. With the development of the computer science, especially the rapid advance of high-performance computing, numerical modelling provides a promising alternative solution for geomechanical researchers. Compared with the theoretical analysis, it is able to consider the complex geometries, the dynamic processes and the nonlinear constitutive responses involved in the related geotechnical phenomena. Compared with the experimental methods, numerical modelling is capable of performing a large number of parameter analyses with reasonable time and relatively low cost. Although many researchers still have doubts on the ability of the numerical modelling to solve practical engineering problems, the numerical simulation seems to be the only feasible solution to understand the complex geotechnical problems. For example, when a geotechnical hazard occurs, the numerical modelling is helpful to find the actual reason through reproducing the observed results with parametric analysis. Actually, Dr. E.T. Brown said that a breakthrough of computational methods for geomechanics is the most promising way to solve various hard problems faced in the actual geotechnical engineering.

Since the 1950s, many computational methods have been developed (Jing, 2003; Zhu and Tang, 2006; He et al., 2014; Lisjak and Grasselli, 2014). The finite element method (FEM) is a typical technology that was firstly developed in the engineering application and then theoretically completed by mathematicians. Its development has experienced the initial popular, the middle trough, and the final mature (a typical growth curve of science and technology). Success of the commercial software of the FEM makes its usage quite convenient. For example, the ANSYS and ABAQUS are widely used in the geomechanics and geotechnical engineering. In addition to the FEM, the discrete element model (DEM) is another well-known computational method in the geomechanics, which was originally developed to solve the progressive failure and movement of rock masses (Cundall, 1971). Now, it has been widely

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used in many disciplines such as material science and chemical engineering (Zhao, 2015). In recent years, the scientific communities are interested in the DEM due to its ability to simulate high complex phenomena. Besides, due to its bottom-up philosophy, the basic principle of the DEM is very intuitive and easy to understand (Chen and Zhao, 1998). Similar to the DEM, the lattice spring model (LSM) is a bottom-up computational method as well. However, its intrinsic limitation on representing the full range of Poisson's ratios (Hrennikoff, 1941) restricts the development of this method. In order to solve this problem, researchers have proposed many solutions, e.g. the multi-body shear spring (Zhao et al., 2011) and the nonlocal potential (Chen et al., 2014). In a recent work, this limitation was further solved by introducing the fourth-dimensional spatial interaction (Zhao, 2017).

In this paper, we explore the ability of the four-dimensional LSM (4D-LSM) on solving specific geomechanical problems. We focus on applying some aspects related to problems in geotechnical mechanics, e.g. the Poisson's ratio, material inhomogeneity, wave propagation, 3D failure, as well as computational efficiency and parallelism. Its ability to naturally model heterogeneous problems is demonstrated through a numerical simulation on a two-phase bar compression problem. Capabilities of the 4D-LSM on stress wave propagation are verified against an analytical solution for the P-wave and S-wave propagations in one-dimensional bar problems. Following this, the parallel computing efficiency and ability on handling 3D fracturing of the 4D-LSM are compared with the parallel distinct LSM (DLSM) (Zhao et al., 2013). Finally, we summarise and discuss the advantages and disadvantages of the 4D-LSM and further possible development.

2. Four-dimensional lattice spring model (4D-LSM)

The original LSM developed by Hrennikoff (1941) is intrinsically limited to solve elastic problems with a fixed Poisson's ratio of 1/3. Its 3D version is only able to handle problems with a fixed Poisson's ratio of 1/4. There are a number of solutions developed and a consensus is reached that noncentral/nonlocal interaction has to be introduced to solve the Poisson's limitation of classical LSM. Nevertheless, recently, it has been shown that it is possible to overcome the Poisson's limitation with only central interaction, but we have to consider the fourth-dimensional interaction. Because our experience is based on the perception of the 3D space, it is hard to directly image the 4D space. The concept of parallel world, a common concept used in science fiction movies, is a straightforward way to explain the idea of the 4D-LSM. In the 4D-LSM, our world is assumed as a hyper-membrane made up from our visual 3D world and an invisible parallel world. Fig. 1 illustrates the process of building a 4D computational model. First, build up a 3D lattice model and assign one additional dimension for each particle. Then, make a copy of this 3D model with an offset along the fourth-dimensional direction (4D thickness). For regular cubic lattice, the lattice configuration can be viewed through a tesseract (a 4D cube). The interaction between two particles is given as

$$F_{ij} = ku_n n_{ij} \tag{1}$$

where k is the spring stiffness, u_n is the deformation of the spring and n_{ij} is the normal vector from particle i to particle j. Compared with classical 3D-LSM, the only difference is that the force and normal vectors have four components. In 4D-LSM, all springs representing the 3D interaction share the same stiffness (k^{3D}), whereas different spring stiffnesses were assigned for the fourth-dimensional interaction, which is characterised by a ratio λ^{4D}. To reproduce the isotropic elasticity, spring stiffnesses of the 4D-LSM have to be assigned through the following equation:

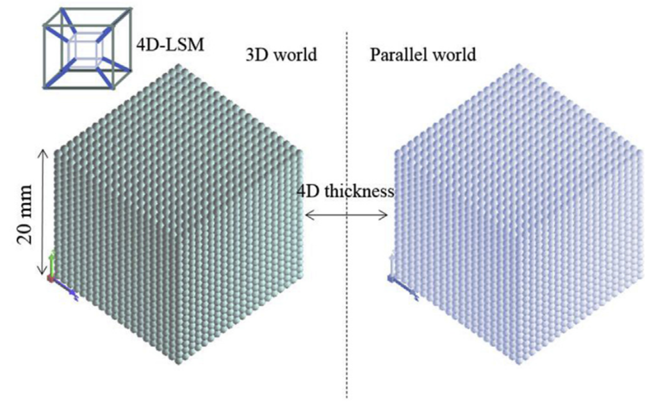

Fig. 1. A regular packed 4D-LSM for the uniaxial compression test to extract the elastic properties.

$$k_\alpha = k_\beta = 4k_\gamma/3 = \lambda^{4D}k^{3D} \tag{2}$$

where k_α, k_β and k_γ are the specific fourth-dimensional stiffnesses. In the 4D-LSM, the central finite difference method is used to solve the system equation, which can be simply written as

$$\ddot{\boldsymbol{x}} = \frac{\boldsymbol{F}}{m} \tag{3}$$

where $\ddot{\boldsymbol{x}}$ is the acceleration, $\boldsymbol{F}$ is the particle force, and m is the particle mass. To obtain the static solution, the local damping scheme can be used (Zhao, 2017). For elastic problems, there are only two parameters, i.e. k^{3D} and λ^{4D}, are needed in the 4D-LSM. The following empirical equations were provided to link the macroscopic elastic parameters (Zhao, 2017):

$$k^{3D} = \frac{6V^{3D}E}{\sum l_{3D,i}^2} \tag{4}$$

$$\lambda^{4D} = -211.13493779\nu^3 + 162.84655851\nu^2 - 55.42449719\nu + 6.92902211 \tag{5}$$

where V^{3D} is the volume of the corresponding represented 3D model, $l_{3D,i}$ is the length of the 3D springs, E is the elastic modulus, and ν is the Poisson's ratio. More details of the 4D-LSM can be found in the work of Zhao (2017).

3. Numerical examples

3.1. Influence of 4D thickness on the elastic prosperities

For the construction of 4D-LSM, we can consider that there is a parallel version of 3D-LSM in the fourth dimension using a parallel world concept. The distance between the 3D model and its parallel version is defined as 4D thickness ratio, which is a ratio of the distance between two models to the lattice length of 3D lattice (regular lattice). As shown in Fig. 1, the model has 8000 particles, whose diameter is 1 mm and its elastic modulus is taken as 10 GPa. A uniaxial compression test is conducted for the model in which the bottom surface is fixed in y direction and a loading velocity of 10 mm/s is applied on the upper surface. The calculation of elastic modulus is expressed as