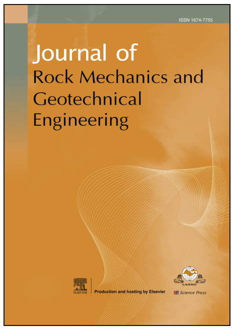

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Luis Fernando Contreras, Edwin T. Brown, Marc Ruest

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Bayesian data analysis to quantify the uncertainty of intact rock strength

Luis Fernando Contreras[a, *], Edwin T. Brown[b], Marc Ruest[a]

[a] *The University of Queensland, St Lucia Campus, Brisbane, QLD 4072, Australia*

[b] *Golder Associates Pty. Ltd., Brisbane, Australia*

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Abstract: One of the main difficulties in the geotechnical design process lies in dealing with uncertainty. Uncertainty is associated with natural variation of properties, and the imprecision and unpredictability caused by insufficient information on parameters or models. Probabilistic methods are normally used to quantify uncertainty. However, the frequentist approach commonly used for this purpose has some drawbacks. First, it lacks a formal framework for incorporating knowledge not represented by data. Second, it has limitations in providing a proper measure of the confidence of parameters inferred from data. The Bayesian approach offers a better framework for treating uncertainty in geotechnical design. The advantages of the Bayesian approach for uncertainty quantification are highlighted in this paper with the Bayesian regression analysis of laboratory test data to infer the intact rock strength parameters σ_{ci} and m_i used in the Hoek-Brown strength criterion. Two case examples are used to illustrate different aspects of the Bayesian methodology and to contrast the approach with a frequentist approach represented by the nonlinear least squares (NLLS) method. The paper discusses the use of a Student's *t*-distribution versus a normal distribution to handle outliers, the consideration of absolute versus relative residuals, and the comparison of quality of fitting results based on standard errors and Bayes factors. Uncertainty quantification with confidence and prediction intervals of the frequentist approach is compared with that based on scatter plots and bands of fitted envelopes of the Bayesian approach. Finally, the Bayesian method is extended to consider two improvements of the fitting analysis. The first is the case in which the Hoek-Brown parameter, *a*, is treated as a variable to improve the fitting in the triaxial region. The second is the incorporation of the uncertainty in the estimation of the direct tensile strength from Brazilian test results within the overall evaluation of the intact rock strength.

Keywords: uncertainty; intact rock strength; Bayesian analysis; Hoek-Brown criterion

1. Introduction

One of the major difficulties encountered by the rock engineer is dealing with the uncertainties present in every aspect of the geotechnical design process. Uncertainty is associated with natural variation of properties, and the imprecision and unpredictability caused by the lack of sufficient information on parameters or models (Baecher and Christian, 2003). Design strategies to deal with the problems associated with uncertainty include conservative design options with large factors of safety, which can be adjusted during the implementation phase based on observations of performance, and the use of probabilistic methods that attempt to measure and account for uncertainty in the design (Christian, 2004).

The probabilistic method commonly used to treat uncertainty in rock mechanics design belongs to the so-called frequentist approach, but this methodology has some drawbacks (VanderPlas, 2014). First, the approach lacks a formal framework to incorporate subjective information such as engineering judgement. Second, it has limitations in providing a proper measure of the confidence of parameters inferred from data. The Bayesian approach provides an alternative route to the conventional probabilistic methods used in geotechnical design; some examples are presented by Miranda et al. (2009), Zhang et al. (2009, 2012), Brown (2012), Bozorgzadeh and Harrison (2014), Feng and Jimenez (2015) and Wang et al. (2016). The approach is based on a particular interpretation of probability and offers an adequate framework for the treatment of uncertainty in geotechnical design.

Probabilistic data analysis using the Bayesian approach involves numerical procedures to estimate parameters from posterior probability distributions. These distributions are the result of combining prior information with available data through Bayes' equation (Kruschke, 2015). The posterior distributions are often complex, multidimensional functions whose analysis requires the use of a class of methods called Markov chain Monte Carlo (MCMC) (Robert and Casella, 2011). These methods are used to draw representative samples of the parameters investigated, providing information on their best estimate values, variability and correlations. The understanding of the concepts behind various algorithms used to perform MCMC analysis is important to properly assess the quality of results. However, the analyst does not have to develop the software in order to use the method. There are already elaborated open source packages in various programming languages (Foreman-Mackey et al., 2013; Smith, 2014; Vincent, 2014) developed by computer scientists and related specialists, which have been tested extensively by these communities. These packages can be easily incorporated into ad-hoc codes for different modelling applications.

This paper presents initially the concepts of geotechnical uncertainty and provides a contrast between the frequentist and Bayesian approaches to quantify uncertainty. The description of the Bayesian approach with reference to the case of the inference of parameters is used to highlight the advantages of this methodology over the frequentist approach. The Bayesian methodology is applied to estimating the intact rock strength parameters σ_{ci} and m_i of the Hoek-Brown strength criterion (σ_{ci} is the UCS of intact rock, and m_i is a constant of the intact rock material), through the analysis of data from compression and tension tests. Two data set examples are presented to compare the Bayesian approach with the nonlinear least squares (NLLS) regression method representing the frequentist approach. The results of these example cases are used to discuss different aspects of the analysis, including the advantages of evaluating errors with a Student's *t*-distribution to handle outliers, the implications of using absolute and relative residuals, and the measure of the quality of the fitting results. The second example is used to emphasise the advantages of the uncertainty quantification with the scatter plots and bands of fitted envelopes of the Bayesian approach, in contrast to the use of confidence and prediction intervals in the frequentist method. Finally, the versatility of the Bayesian method is illustrated with two situations that require the model to be extended to include additional parameters for inference. The first case corresponds to the consideration of the Hoek-Brown parameter, *a*, as a free variable so that the fitting in the triaxial compression region is not constrained by that obtained in the tensile and uniaxial compression regions based on a two-parameter model. The second case is the inclusion of the uncertainty in the conversion from Brazilian tensile strength

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*Corresponding author. E-mail address: luis.contreras@uq.net.au