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Multistable two-dimensional spring-mass lattices with tunable band gaps and wave directionality

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ABSTRACT

In this paper, we consider elastic wave propagation in two dimensional periodic lattices that include an alternating pattern of linear springs and nonlinear bistable springs. Because of the bistable springs, these lattices have multiple stable equilibrium configurations. An analytical model that takes into account both nonlinear geometrical effects and nonlinearity of the springs is developed for the total potential energy of the system. After finding the stable equilibrium configurations of the unit cell by minimizing its potential energy, the propagation of waves of small amplitude is analyzed in each of these stable configurations using Bloch theorem. Examination of the band diagrams demonstrates that, depending on the model parameters, directional or complete band gaps can be observed in some of the deformed configurations, particularly for deformed configurations that are close to a critical point. Furthermore, analysis of the iso-frequency contours of the dispersion surfaces and of polar plots of the phase and group velocities indicates that the low frequency wave directionality of the lattice is tunable. For some designs, switching from one configuration to another configuration makes it possible to dramatically alter the preferred direction of wave propagation. Simulations of the response of lattices of finite size confirm that these lattices can be used as reconfigurable phononic crystals with tunable directivity, both in the low frequency range and within the directional band gaps.

1. Introduction

Phononic crystals and acoustic metamaterials are materials that have the ability to filter, control or guide the propagation of elastic waves [1] due to their well-defined architecture, which is typically periodic for ease of analysis and fabrication. For example, some phononic crystals exhibit frequency ranges, called band gaps, where the propagation of waves is prohibited [2]. Furthermore, phononic crystals and acoustic metamaterials can be used to guide the propagation of elastic waves along preferred directions [3,4]. The directionality of wave propagation has been particularly studied in two-dimensional (2D) lattice materials using spring-mass lattice [5,6], beam [3,7,8], and continuum [9] models. These studies have shown that the preferred direction of wave propagation is highly dependent on the geometry [8] and symmetry of the unit cell [10].

The functionality of phononic crystals and acoustic metamaterials can be enhanced if their properties can be tuned on demand. Many strategies have been proposed to tune the phononic band gaps and wave directionality of phononic crystals and acoustic metamaterials. For example, applying deformation can be used to reversibly tune the band gaps of periodic elastomeric structures due to elastic instabilities that cause configurational changes in the architecture [11–13]. Deformation also causes the emergence of directional band gaps [14] and changes in the directionality and anisotropy of periodic structures [15]. The

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effect of large deformation on the wave properties of mass-spring hexagonal lattices has been analyzed [6]; numerical results have demonstrated that the wave propagation response can vary from isotropic to highly directional depending on the preload. The analysis of the tunability of the band gaps and wave directionality has been extended to hierarchical honeycomb lattices by Mousanezhad et al. [16]. Other approaches, based on the use of active elements, have also been proposed. For example, piezoelectric elements can be used to reconfigure the cell symmetry of cellular metamaterials in order to tune the directionality [17]. It is possible to use tunable piezoelectric electromechanical resonators to reversibly override the intrinsic anisotropy of square lattices [18]. Magneto-elastic lattices have also highly tunable band gaps and wave directionality [19].

In this paper, we propose to tune the directionality of 2D lattices using an alternative approach, based on the use of bistable elements. The design, fabrication and quasi-static mechanics of metamaterials that consist of multiple bistable elements that are arranged in series have been extensively studied in recent years [20–26]. Truly two-dimensional or three-dimensional metamaterials and deployable metastructures with more general arrangements of bistable elements have also been designed [24,27]. Multistable metamaterials have excellent energy absorption and damping properties [20,22,26]. Furthermore, design strategies can be developed to synthesize multistable metastructures and metamaterials that have highly adaptive stiffness and/or hysteretic damping properties [28,29]. In addition to attractive quasi-static properties, multistable systems have interesting properties in response to dynamic loads [30–34]. For example, the unidirectional propagation of transition waves [31], the ability of waves to propagate over large distances despite the presence of dissipation mechanisms [32], or nonreciprocal wave propagation [35] can be observed in response to large amplitude stimuli. The propagation of transition waves can be exploited to generate energy harvesting capabilities that do not depend on the amplitude or frequency of the input [36]. Reconfiguration can also be harnessed to alter the propagation of waves of small amplitudes. For example, we have recently shown that switching from one stable configuration to another stable configuration can be exploited to obtain tunable band gaps in 1D chains of bistable elements [34]. Goldsberry and Haberman [37] have demonstrated that honeycombs with negative stiffness exhibit highly tunable directionality depending on the prestrain. However, the studies of Refs. [34,37] were limited to systems in which the bistable or negative stiffness elements are arranged in series. The effect of switching from one stable configuration to another stable configuration in more general, truly 2D systems with bistable elements has not been analyzed.

The objective of this work is to demonstrate that bistable elements can be exploited to reversibly tune elastic wave propagation in 2D spring-mass lattices. Analytical models for the total potential energy of a unit cell are developed in order to determine the stable configurations of the system and to analyze wave propagation in each of the stable configurations. In order to illustrate how switching from one configuration to another configuration can be exploited to significantly tune the wave directionality, two different types of square lattices are analyzed: (1) lattices with horizontal, vertical and diagonal springs (2) lattices with horizontal and vertical springs as well as rotational (torsional) springs at the joints.

2. Methods

2.1. 2D lattice with linear and bistable springs

Except for the presence of bistable springs and rotational springs, the lattices that are proposed here have a similar topology as the mass-spring square lattices analyzed by Jensen [5]. The systems consist of a 2D lattice made of an alternating pattern of linear springs, of stiffness k_0, and bistable springs (Fig. 1). Because of this alterning pattern, the unit cell consists of four different masses of position vector $\mathbf{x}_i = (x_i, y_i)$, where $i = 1, .., 4$. Each mass has two degrees of freedom: its horizontal displacement, u_i, and its vertical displacement, v_i. In the undeformed configuration, all horizontal and vertical springs have the same stiffness, k_0.

To eliminate zero-energy deformation modes, additional springs need to be added to the lattice. Two different designs were considered: in Design D, shown in Fig. 1(a)-(b), diagonal springs of stiffness $k_t = \alpha_t k_0$ (where α_t is a non-dimensional parameter) are included within each unit cell; in Design R, shown in Fig. 1(c)-(d), rotational springs of stiffness $k_R = \alpha_R k_0 L^2$ (where α_R is a non-dimensional parameter and L is the undeformed length of the horizontal and vertical springs) are located at the connection between two neighboring edges. While the physical principles for the tunability of wave propagation are similar for these two designs, the numerical results shown in the remainder of this paper demonstrate that there are important differences in how elastic waves propagate for these two lattice designs.

The potential energy of a linear spring that connects mass i to mass j can be written in the following form:

$$U^L_{ij} = \frac{1}{2} k_0 \Big(\|\mathbf{x}_j - \mathbf{x}_i\| - \left\|\mathbf{x}^0_j - \mathbf{x}^0_i\right\| \Big)^2, \quad (1)$$

where $\mathbf{x}^0_i$ and $\mathbf{x}^0_j$ are the initial position vectors of mass i and j, respectively; $\|.\|$ denote the norm of the vector. The potential energy of a rotational spring between the edge ij that connects node i and j and the edge ik that connects node i to node k is:

$$U^R_{ij,ik} = \frac{1}{2} k_R \Delta\theta^2_{ij,ik}, \quad (2)$$

where $\Delta\theta_{ij,ik}$ is the change in the value of the angle between edge ij and edge ik from its initial value.