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Sound radiation of a vibrating elastically supported circular plate embedded into a flat screen revisited using the Zernike circle polynomials

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A B S T R A C T

This study deals with the problem of sound radiation by an elastically supported thin circular plate. The plate is excited asymmetrically. In such cases, usually different approximate methods are used to calculate the acoustic power. These methods are time-consuming, and some of them are applicable only to axisymmetric problems. They work only for the lowest and the highest frequencies. Consequently, their applications are limited. Another shortcoming of such methods is that they work only for either clamped circular plates or simply supported plates. In practical applications, the boundary conditions of circular plates are often required. Therefore, the model of an elastically supported plate is of a great interest in this regard. Finally, the methods presented in the literature are time-consuming and not highly accurate. They often incorporate numerical integration to calculate the acoustic pressure and the acoustic power. The use of the radial polynomials leads to much more accurate and efficient results. The acoustic power is expressed in terms of the modal impedance coefficients. The coefficients are calculated without numerical integration and with arbitrary precision in a wide low-frequency band. They are expressed in terms of a rapidly converging expansion series. The formulas presented are useful to solve the problem of vibration of a plate including arbitrary excitation, material damping, and fluid-structure interactions. This results in a system of three coupled differential equations. The first one is the Helmholtz equation governing the fluid vibrations in the upper half-space, the second one is also the Helmholtz equation governing the fluid vibrations in the lower half-space, and the third one deals with the motion of the plate. Consequently, it leads to a system of algebraic equations. Finally, the effects of the plate's boundary conditions on the acoustic power, the far field, and the near field are analyzed numerically.

1. Introduction

Sound radiation by a vibrating circular plate is a classical problem. Many researchers have worked on this problem using both theoretical and experimental methods. The results obtained are of a practical importance since they can be used in different areas of physics and engineering. Vibrating, sound radiating, and sound sensing circular plates are used in various electro-acoustic transducers, electrostatic speakers, earphones, microphones, covers, etc. The vibrations of such plates have been addressed separately in the literature by different authors such as Rayleigh [1], Leissa [2], Meirovitch [3], Rao [4], and

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investigated in great details [5–8]. The well-known Chladni's plates have been under constant focus for many years [9–23]. Investigating Chladni's plates has a great practical importance for finding the resonance frequencies of flat plates, resonators, and musical instruments.

The solutions to the vibration problems of circular plates constitute the base for further analysis of sound radiations of such sources. Vibration problems are also investigated in parallel with fluid-structure interactions. The foundations for sound radiation of flat sources are given in the textbooks by Rayleigh [24], Skudrzyk [25], Morse and Ingards [26], Kuttruff [27], Kleiner [28], Beranek [29], and Beranek and Mellow [30]. Rayleigh [24] (cf. Sec. 278) presented his famous first integral for the acoustic pressure of a planar vibrating source (cf. also Williams [31] Eq. (2.75) p. 36). There are also numerous papers describing the problems of sound radiation from vibrating circular plates. King [32] analyzed sound radiation of a flat circular piezo-electric transducer. He provided some useful integral expressions for the acoustic pressure, which are named after him as King's integral. This is, in fact, Hankel's representation of Rayleigh's first integral. Levine and Leppington [33] presented asymptotic formulas for the effective attenuation coefficient of a vibrating circular plate. Farstad et al. [34] discussed in detail the sound radiation efficiency of an elastically supported rigid rotor joined to a flexible annular disk. Hasheminejad and Afsharmanesh [35] analyzed vibroacoustic behavior of a vibrating sandwich annular disk. Leniowska and Mazan [36] experimentally investigated active vibration control of a vibrating clamped circular plate. Hasheminejad and Keshavarzpour [37] analyzed vibroacoustic behavior of a vibrating composite circular plate. Shakeri and Younesian [38] investigated the steady state and transient vibraoacoustic responses of an excited annular plate with a tuned mass damper. Chiang and Huang [39] dealt with the resonance frequencies and acoustic pressure radiated by circular electrostatic and piezoelectric transducers. Rdzanek et al. [40] presented the asymptotic formulas for the acoustic power of a single axisymmetric dominant mode of a simply supported circular plate. Aarts and Janssen [41] analyzed sound radiation from a resilient disk vibrating axisymmetrically. They presented a highly efficient and accurate expansion series for the modal acoustic impedance coefficients using the radial polynomials. Rdzanek [42] applied the same method to find the acoustic power output of a clamped circular plate excited asymmetrically. Some more complex fluid-structure interactions were also investigated [43–52].

To the best of author's knowledge, the problem of sound radiation from an elastically supported thin circular plate excited asymmetrically has not yet been solved using the radial polynomials. In this paper, such a solution is presented and the results are highly accurate and efficient.

The study is organized as follows. The solution to the Helmholtz equation is presented in Sec. 2. The eigenfunction expansion of the plate, Cerjan's expansion, and Hankel's transforms are used for this purpose. The problem of free vibrations of an elastically supported plate is presented including the frequency equation. The acoustic pressure and the directivity pattern are obtained on the basis of Green's function for both half-spaces. The Neumann boundary condition is also applied for this purpose. The modal impedance coefficients are calculated accurately in terms of a highly convergent hypergeometric series. The problem of the asymmetrically excited vibrations of the plate is solved including the fluid loading effect and the plate's material damping. The three most representative types of excitation such as the point excitation, annular sector excitation, and circle excitation are considered. All the physical quantities are analyzed numerically focusing on the effect of the plate's boundary conditions on the resultant acoustic pressure and the acoustic power. The acoustic pressure is calculated using Rayleigh's first integral. The numerical analysis is presented in Sec. 3 and the concluding remarks in Sec. 4. Appendix A contains some useful integrals. Appendix B presents the rigorous validation of the results obtained. Appendix C explains how Graf's theorem is used to obtain the modal circle excitation coefficients. Appendix D shows how the shifted and scaled Zernike circle polynomials are introduced. Appendix E presents the usage of the plate's boundary conditions to simplify some expressions. Appendix F contains formulas such as the frequency equation and the eigenfunction coefficients in the major specific boundary configurations such as for the free, clamped, guided and simply supported plates. Appendix G explains how to apply the dominant mode simplification.

2. Governing equations

2.1. The boundary value problem formulation

A thin elastically supported circular plate of the radius a is embedded into a flat rigid screen (cf. Figs. 1 and 2). The plate vibrates with the normal component of velocity $v(\vec{r},t) = v(\vec{r})\exp(-\mathrm{i}\omega t)$ (m s^{-1}), where $v(\vec{r})$ is the amplitude, $\vec{r} = (r,\phi,z)$ (m) is the radius vector of a spatial point in the cylindrical coordinates for $0 \leqslant r < \infty$ (m), $-\pi \leqslant \phi' \leqslant \pi$ (rad), and $0 \leqslant |z| < \infty$ (m), $\vec{r}\,' = (r',\phi',0)$ (m) is the radius vector of the excitation center in the cylindrical coordinates for $0 \leqslant r' < a$ (m) and $-\pi \leqslant \phi' \leqslant \pi$ (rad), $\vec{r} = (|\vec{r}\,|,\theta,\phi)$ (m) is the radius vector of a spatial point in the spherical coordinates for $0 \leqslant |\vec{r}\,| < \infty$ (m), $0 \leqslant \theta \leqslant \pi$ (rad), and $-\pi \leqslant \phi' \leqslant \pi$ (rad), $\exp(-\mathrm{i}\omega t)$ is the time dependence, $\omega = kc$ (rad s^{-1}) is the angular frequency, $k = 2\pi/\Lambda$ (rad m^{-1}) is the wavenumber, Λ (m) is the wavelength, c (m s^{-1}) is the speed of sound, and t (s) is the time variable. The space above and below the screen is filled with a fluid of the ambient density ϱ_0 (kg m^{-3}). Consequently, the plate radiates acoustic pressure p_1 into upper half space $\Omega_1 = \{z > 0\}$ and acoustic pressure p_2 into lower half space $\Omega_2 = \{z < 0\}$. The following Helmholtz equation is satisfied in both half spaces

$$\left(\nabla^2 + k^2\right) p_\mu(\vec{r},t) = 0, \tag{1}$$