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Internal resonances and modes interactions in non-linear vibrations of viscoelastic heterogeneous solids

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A B S T R A C T

The aim of the paper is to study how viscous damping influences mode coupling in non-linear vibrations of microstructured solids. As an illustrative example, natural longitudinal vibrations of a layered heterogeneous medium are considered. The macroscopic dynamic equation is obtained by asymptotic homogenisation. The input continuous problem is analysed using a spatial discretisation procedure. An asymptotic solution is developed by the method of multiple time scales and the fourth-order Runge-Kutta method is employed for numerical simulations. Internal resonances and energy transfers between the vibrating modes are predicted and analysed. The conditions for possible truncation of the original infinite system are discussed. The obtained numerical and analytical results are in good agreement.

1. Introduction

Real materials and structures are never perfectly elastic. In dynamic problems, part of the mechanical energy is always transformed to heat because of the internal friction. Dissipative behaviour of heterogeneous solids can be determined by various factors, such as properties of the components, features of the microstructure, bonding conditions at the "matrix-inclusion" interface, etc. [1–4]. Here we address the case when energy dissipation is caused by the viscoelasticity of the components, which is typical, for example, polymer-based materials.

Viscoelastic properties of the medium can be described by the well known Kelvin-Voigt model (see, for example [5]). In hydrodynamics, it corresponds to the classical behaviour of a viscous gas, where shear stresses are proportional to the deformation rates and the proportionality coefficients are determined by the gas density. For solids, the Kelvin-Voigt model can be naturally deduced from a lattice-type model through passing to the continuous limit and assuming that the interaction forces between neighbouring particles depend on the rate of the distance change, rather than on the distance itself [4].

Pal'mov [6,7] studied non-linear vibrations of semi-infinite and finite rods subjected to a dissipation. He employed the method of harmonic linearisation, which gave first order approximation coinciding with the Rayleigh-Ritz approach.

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Equations in slow variables were obtained and analysed. High-frequency deformations of non-linear rate dependent materials induced by the propagation of transient waves were considered by Varley and Rogers [8] and Seymour and Varley [9]. Mortell and Varley [10] discussed non-linear elastic waves in finite-size bodies and studied the dynamic response of a viscoelastic rod subjected to a pulse load [11]. In the first order approximation, the solution was developed as a superposition of two modulated waves travelling in opposite directions and not interacting with each other.

Vibrations of continuous structures can be described by dynamical systems having infinite degrees of freedom. Non-linearity leads to localisation of energy and its transfer from the low- to the high-frequency part of the spectrum and vice versa. The vibration modes can then be involved in complicated interactions, resulting in internal resonances and self-generation of higher-order modes. In such a case, truncation to the modes having non-zero initial energy (which is usually applicable in linear problems) is not possible and all the resonant modes should be taken into account.

The effects of mode coupling and internal resonance have attracted the considerable attention of many authors. A number of results were obtained for vibrations of homogeneous structures and numerical [12] as well as asymptotic [13–17] approaches have been widely applied. However, the non-linear dynamic behaviour of heterogeneous solids was studied to a significantly less extent. Only recently, one-dimensional vibrations of microstructured rods were considered by Andrianov et al. in respect of a non-linear elastic external medium [18] and geometrical and physical non-linearity [19]. Heterogeneity results in dispersion of energy and, thus, compensates the influence of non-linearity. As the spatial period of the modes decreases and approaches the size of the microstructure, internal resonances are suppressed and truncation to only a few leading order modes can be justified.

In our previous works we omitted the influence of viscous damping. It should be noted that dissipation restrains energy transfers between the vibration modes and on a large time scale modes coupling vanishes. In that sense, dispersion and dissipation acting in a non-linear system may lead to qualitatively similar physical consequences (Zabusky and Kruskal encountered this analogy analysing the FPU problem, see Ref. [20]).

In this paper, non-linear vibrations of a layered viscoelastic solid are studied. We propose a macroscopic wave equation describing the dynamic behaviour of a body taking into account the properties of the microstructure. Through use of the developed model, the interplay between the effects of non-linearity and dissipation is analysed. We aim to predict how viscous damping influences modes coupling and justify truncation of the original infinite system to a finite number of the leading order modes.

The paper is organised as follows. In Section 2, the input constitutive relations are introduced and the homogenised dynamical equation formulated. In Section 3, the discretisation procedure is presented and two different types of the mode interactions discussed. In Section 4, natural vibrations are studied by the method of multiple time scales. Section 5 is devoted to the numerical simulation of modes coupling and comparison of the numerical and asymptotic solutions. Concluding remarks are presented in Section 6.

2. Input problem and homogenised dynamical equation

Let us consider a heterogeneous solid consisting of periodically repeated viscoelastic layers $\Omega^{(1)}$ and $\Omega^{(2)}$ (Fig. 1). We shall study natural longitudinal vibrations in the direction x. This model can describe the properties of laminated composite materials, phononic crystals and acoustic diodes (see, for example [21–23]). Layered structures are employed in band-gap engineering for the design of vibration and sound control devices [24].

The mechanical behaviour of each of the layers $\Omega^{(i)}$ is described by the Kelvin-Voigt model [5], which includes a purely elastic spring connected in parallel with a viscous dashpot. The properties of the dashpot are assumed to be linear, while the spring exhibits non-linear response. Geometric non-linearity is taken into account using the Cauchy–Green strain tensor [25]

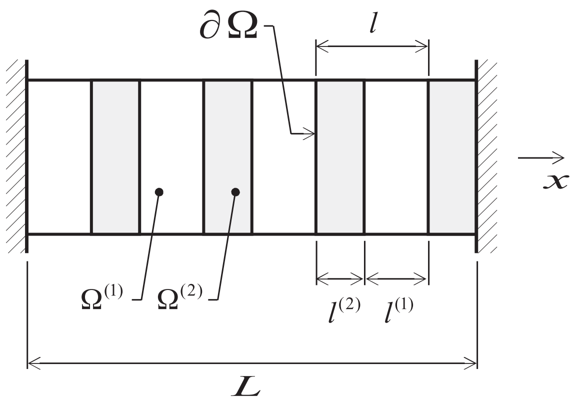

Fig. 1. Periodically heterogeneous solid under consideration.