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Wiener Path Integral based response determination of nonlinear systems subject to non-white, non-Gaussian, and non-stationary stochastic excitation

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ABSTRACT

The recently developed Wiener Path Integral (WPI) technique for determining the joint response probability density function of nonlinear systems subject to Gaussian white noise excitation is generalized herein to account for non-white, non-Gaussian, and non-stationary excitation processes. Specifically, modeling the excitation process as the output of a filter equation with Gaussian white noise as its input, it is possible to define an augmented response vector process to be considered in the WPI solution technique. A significant advantage relates to the fact that the technique is still applicable even for arbitrary excitation power spectrum forms. In such cases, it is shown that the use of a filter approximation facilitates the implementation of the WPI technique in a straightforward manner, without compromising its accuracy necessarily. Further, in addition to dynamical systems subject to stochastic excitation, the technique can also account for a special class of engineering mechanics problems where the media properties are modeled as stochastic fields. Several numerical examples pertaining to both single- and multi-degree-of-freedom systems are considered, including a marine structural system exposed to flow-induced non-white excitation, as well as a bending beam with a non-Gaussian and non-homogeneous Young's modulus. Comparisons with Monte Carlo simulation data demonstrate the accuracy of the technique.

1. Introduction

Uncertainty propagation in engineering mechanics and dynamics is a highly challenging problem that requires development of analytical/numerical techniques for determining the stochastic response of complex engineering systems. In this regard, although Monte Carlo simulation (MCS) has been the most versatile technique for addressing the above problem (e.g., [1,2]), it can become computationally daunting when faced with high-dimensional systems or with computing very low probability events. Thus, there is a demand for pursuing more computationally efficient methodologies. In the field of stochastic engineering dynamics, a number of alternative techniques, such as stochastic averaging (e.g., [3–5]), statistical linearization (e.g., [6–8]), as well as methodologies based on Markov approximations and related Fokker-Planck equations [9], have been developed over the

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past few decades with varying degrees of accuracy.

More recently, a Wiener Path Integral (WPI) technique, whose origins can be found in theoretical physics [10], has been developed in the field of engineering dynamics for determining the response transition probability density function (PDF) of oscillators subject to Gaussian white noise excitation [11]. The technique has been generalized to account for multi-degree-of-freedom (MDOF) systems and diverse nonlinear/hysteretic system modeling [12], as well as for systems endowed with fractional derivative terms [13]. Further, the technique has been enhanced from a computational efficiency perspective by relying on its localization capabilities and invoking appropriate expansions for the response PDF [14]. In this regard, sparse PDF representations in conjunction with compressive sampling tools and group sparsity concepts have been utilized in Ref. [15] for addressing relatively high-dimensional stochastic systems. Finally, it has been shown by Kougioumtzoglou [16] that the technique can also address a special class of engineering mechanics problems where media properties are modeled as stochastic fields, while preliminary efforts on quantifying the error of the technique can be found in Ref. [17]. Nevertheless, the WPI technique has been limited so far to treating Gaussian white noise excitation processes only.

In this paper, the WPI technique is extended to account for non-white, non-Gaussian and non-stationary processes representing either the excitation of an MDOF dynamical system, or the media properties of a class of one-dimensional continuous systems. To this aim, modeling the excitation process as the output of a filter equation with Gaussian white noise as its input (e.g., [18]), it is possible to define an augmented response vector process to be considered in the WPI solution technique. A significant advantage relates to the fact that the technique is still applicable even for arbitrary excitation power spectrum forms. In such cases, it is shown that the use of a filter approximation (see also [19]) facilitates the implementation of the WPI technique in a straightforward manner. Several numerical examples pertaining to both single- and multi-degree-of-freedom systems are considered, including a marine structural system exposed to flow-induced non-white excitation, as well as a bending beam with a non-Gaussian and non-homogeneous Young's modulus. Comparisons with MCS data demonstrate the accuracy of the technique.

2. Preliminaries

2.1. Fokker-Planck equation

This section serves as a brief background on Markov processes, the associated Chapman-Kolmogorov (C-K) and Fokker-Planck (F-P) equations, as well as their relation to a corresponding stochastic differential equation (SDE).

Consider a Markov stochastic vector process, $\boldsymbol{\alpha}(t)$, where $\boldsymbol{\alpha} = [\alpha_j]_{n\times 1}$, for which the C-K equation is satisfied (e.g., [20]) for any $t_{l+1} \geq t_l \geq t_{l-1}$, i.e.,

$$p(\boldsymbol{\alpha}_{l+1}, t_{l+1} \mid \boldsymbol{\alpha}_{l-1}, t_{l-1}) = \int_{-\infty}^{\infty} p(\boldsymbol{\alpha}_{l+1}, t_{l+1} \mid \boldsymbol{\alpha}_l, t_l) p(\boldsymbol{\alpha}_l, t_l \mid \boldsymbol{\alpha}_{l-1}, t_{l-1}) \mathrm{d}\boldsymbol{\alpha}_l \tag{1}$$

where $p(\boldsymbol{\alpha}_{l+1}, t_{l+1} \mid \boldsymbol{\alpha}_{l-1}, t_{l-1})$ denotes the transition PDF of the process $\boldsymbol{\alpha}$. For a Markov process, the sample paths are continuous functions of t with probability one, if the Lindeberg condition is satisfied (e.g., [21]), namely for any $\epsilon > 0$

$$\lim_{\Delta t\to 0} \frac{1}{\Delta t} \int\limits_{|\boldsymbol{\alpha}_{l+1}-\boldsymbol{\alpha}_l|>\epsilon} p(\boldsymbol{\alpha}_{l+1}, t_{l+1} \mid \boldsymbol{\alpha}_l, t_l) \mathrm{d}\boldsymbol{\alpha}_{l+1} = 0 \tag{2}$$

where $\Delta t = t_{l+1} - t_l$. Such a process is called a diffusion process and the components of its drift vector, $\mathbf{A}(\boldsymbol{\alpha}_l, t_l) = [A_j(\boldsymbol{\alpha}_l, t_l)]_{n\times 1}$ and of its diffusion matrix $\mathbf{B}(\boldsymbol{\alpha}_l, t_l) = [B_{jk}(\boldsymbol{\alpha}_l, t_l)]_{n\times n}$ can be defined as (e.g., [22])

$$A_j(\boldsymbol{\alpha}_l, t_l) = \lim_{\Delta t\to 0} \frac{\mathbb{E}\left[\alpha_{jl+1} - \alpha_{jl}\right]}{\Delta t} \tag{3}$$

and

$$B_{jk}^2(\boldsymbol{\alpha}_l, t_l) = \lim_{\Delta t\to 0} \frac{\mathbb{E}\left[(\alpha_{jl+1} - \alpha_{jl})(\alpha_{kl+1} - \alpha_{kl})\right]}{\Delta t} \tag{4}$$

respectively. Further, employing the C-K Eq. (1) leads to the well-known F-P equation (e.g., [23,24])

$$\frac{\partial p}{\partial t} = -\sum_j \frac{\partial}{\partial \alpha_j}\left(A_j(\boldsymbol{\alpha}, t)p\right) + \frac{1}{2}\sum_{j,k} \frac{\partial}{\partial \alpha_j}\frac{\partial}{\partial \alpha_k}\left(\widetilde{B}_{jk}(\boldsymbol{\alpha}, t)p\right) \tag{5}$$

where $p = p(\boldsymbol{\alpha}_{l+1}, t_{l+1} \mid \boldsymbol{\alpha}_l, t_l)$ and $\widetilde{\mathbf{B}}(\boldsymbol{\alpha}, t) = \mathbf{B}(\boldsymbol{\alpha}, t)\mathbf{B}^T(\boldsymbol{\alpha}, t)$.

The F-P Eq. (5) is related to a first-order SDE of the form

$$\dot{\boldsymbol{\alpha}} = \mathbf{A}(\boldsymbol{\alpha}, t) + \mathbf{B}(\boldsymbol{\alpha}, t)\boldsymbol{\eta}(t) \tag{6}$$

where the dot above a variable denotes differentiation with respect to time t and $\boldsymbol{\eta}(t)$ is a zero-mean and delta-correlated process of intensity one; i.e., $\mathbb{E}\left[\eta_j(t)\right] = 0$ and $\mathbb{E}\left[\eta_j(t_l)\eta_k(t_{l+1})\right] = \delta_{jk}\delta(t_l - t_{l+1})$, for any $j, k \in \{1, \ldots, n\}$, where δ_{jk} is the Kronecker delta, and $\delta(t)$ is the Dirac delta function.