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Numerical construction of impulse response functions and input signal reconstruction

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ABSTRACT

A new method for input signal reconstruction is presented. This approach utilizes the convolution relationship of inputs and outputs of linear systems. A linear discretization of sampled points was assumed in formulating the discrete convolution integral. Subsequently, the resulting equation was modified via a linear constraint to facilitate solution by the least square method. This improves the conditioning of discrete deconvolution. The method was validated numerically on a single degree of freedom dynamic system. Inputs reconstructed matched the applied input very well for a low noise case. A methodology for multiple inputs and outputs was developed. The single input-multiple output formulation was validated using experimental strain measurements of hammer pulse tests. Pulse areas and peak magnitudes were reconstructed with good accuracy.

1. Introduction

Inverse signal reconstruction is necessary in a number of physical problems. One often desires knowledge of a system that cannot be directly measured or accurately modeled. Many such linear cases can be represented by the convolution integral which relates the measured output as a function of the desired input and the impulse response of the system. Deconvolution may be applied to reconstruct the desired input. Due to the nature of this formulation, these problems are characterized as inverse problems [1]. Acoustic reverberation reduction [2], surface heat flux measurement [3], and dynamic load measurement, are a few examples of such problems.

The motivation for the present work is dynamic load measurement, primarily for use in hypersonic wind tunnel experiments. There are a host of wind tunnel test scenarios that would benefit from dynamic load measurement capability. Examples include shroud separation [4,5], store separation [6–8], and jet-flow interaction [9–11]. There exist methods that attempt to account for inertial contributions by adding acceleration measurement [12–15]. Other methods use deconvolution of the impulse response function to determine the applied loads [16–18]. Such methods typically utilize regularization to solve the ill-posed inverse problem [19].

Sanchez et al. [20] presents a comprehensive review of many of the force reconstruction techniques employed to date. The authors categorize the solution methodologies as Direct, Regularization, and Probabilistic/Statistical methods. The Direct methods tend to fit the data to explicit models, e.g. structural models [21] and interpolation functions [22–24]. These approaches have achieved success without the use of regularization but require complex, application specific models or extensive curve fitting. The methods that utilize common regularization schemes, e.g. Tikhonov or TSVD, typically have the added challenge

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of selecting an optimal regularization parameter and may impart unwanted smoothness to the solution. Schuster et al. [25] presents an even more recent review of some of techniques being applied to the broader field of inverse problem solution, much of which focuses on the selection of a regularization parameter [26,27]. Probabilistic/Statistical methods appear to be similarly spirited to Direct methods and often employ a system model for which parameters are to be fit, e.g. structural frequencies and damping of a beam, with the addition of a noise term. This modeling is not desirable and often, regularization or filtering is required to ensure the well-posedness of the system.

In this paper we present a novel method for the solution of ill-posed inverse problems without the use of regularization, filtering, system parameter estimation, or curve fitting. We seek a solution methodology that does not impart arbitrary smoothness to counter the ill-posedness. The discussion in the following sections presents the technical approach for the method and provides numerical and experimental examples for validation. Numerical studies are performed on a single input single output (SISO) single degree of freedom (SDOF) spring-mass-damper system. This method is extended to single input multiple output (SIMO) and multiple input multiple output (MIMO) systems. SIMO system validation is performed using data obtained by experiments from a typically sized test article in the Arnold Engineering Development Complex's (AEDC) Balance Calibration Laboratory. This facility is located in White Oak, Maryland and is most well known for its hypervelocity wind tunnel, Tunnel No. 9.

2. SISO formulation and validation

First, we will consider a SISO linear time invariant (LTI) system.

2.1. SISO formulation

For LTI systems, the relationship between an input, $u(t)$, and output, $y(t)$, can be described by the convolution of the input and impulse response function (IRF), $h(t)$, of the system. This relationship can be expressed as

$$y(t) = \int_0^t h(t-\tau)u(\tau)\mathrm{d}\tau. \tag{1}$$

Typical solution of the inverse problem using Eq. (1) invokes the assumption that system response to a finite pulse is approximately the impulse response [16]. If the response is scaled by the area and time shifted by the width of the impulse, this is a decent assumption. Any output response can be deconvolved with this impulse response to reconstruct the input. However, there are two main drawbacks of this: hammer pulses are the only applicable input calibration forces and the deconvolution is ill-posed. In the present paper we will introduce a methodology to remedy these shortcomings and use more of the information collected during a typical calibration. First, one may invoke the commutative property of the convolution integral and rewrite Eq. (1) as

$$y(t) = \int_0^t u(t-\tau)h(\tau)\mathrm{d}\tau. \tag{2}$$

For discrete measurement signals the integral in Eq. (2) can be segmented into a summation of integrals over each pair of sampling points:

$$y(t_k) = \sum_{i=1}^{k-1} \int_{t_i}^{t_{i+1}} u(t_k-\tau)h(\tau)\mathrm{d}\tau. \tag{3}$$

Many discrete representations of the convolution integral utilize the assumption that a sampled point holds constant until the next point is sampled. However, here we will assume that $u(t)$ and $h(t)$ are piecewise linear over the sampling time segment, Δt_s, as follows:

$$u(t) = (1-s)u_i + su_{i+1} \quad t \in [t_i, t_{i+1}] \tag{4a}$$

$$h(t) = (1-s)h_i + sh_{i+1} \quad t \in [t_i, t_{i+1}] \tag{4b}$$

$$s = \frac{t-t_i}{\Delta t_s} \quad t \in [t_i, t_{i+1}] \tag{4c}$$

From Eq. (4c), we can see that $\mathrm{d}t = \Delta t_s \mathrm{d}s$. Note that for convenience of formulation, we have assumed the sampling frequencies to be the same for $u(t)$ and $h(t)$. However, we will subsequently use a larger time segment for $h(t)$ through a linear constraint. Applying this constraint will significantly improve the conditioning of the problem, for solution via the least squares method. For compactness, the expressions in Eq. (4) may be written in matrix form as follows:

$$u(t) = \begin{bmatrix} u_i & u_{i+1} \end{bmatrix} \begin{Bmatrix} N_1 \\ N_2 \end{Bmatrix} \quad t \in [t_i, t_{i+1}] \tag{5a}$$