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Unbalance and resonance elimination with active bearings on general rotors ☆

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ABSTRACT

Unbalances in rotating machinery cause vibration, noise and wear. Active bearings allow the use of specialized control algorithms which eliminate the bearing forces caused by unbalances. Most algorithms suffer from drawbacks, namely their lack of general stability, the need for exact rotor models, or their unclear rotordynamic interactions.

In this work, we found a closed-form analytical solution for the elimination of unbalance-induced bearing forces on arbitrary, gyroscopic rotors. We demonstrate that the force-free condition is met for any unbalance distribution. Furthermore, we proved that up to two resonances can be fully eliminated. We found a new Lyapunov stability theorem to prove the controller's superior stability properties. The advantage of our approach is that different active bearing technologies are unified in a single, generalized theory. Our theory is not only limited to rotors, but applies to all systems with harmonic excitation that share the same general matrix structure. Finally, we provide evidence that our theoretic assumptions are also satisfied in reality: In an experiment we demonstrate that unbalance-induced bearing forces and rotor resonances can not only be eliminated in theory, but also in practice.

1. Introduction

The significance of rotating machinery in modern civilizations is reflected in its intense research for the past 150 years. After the initial discovery that critical bending modes can be passed both practically by Laval and theoretically by Jeffcott [1] and the explanation of the gyroscopic effect by Stodola [2], faster machines emerged which soon led to rotor instabilities. Newkirk and Kimball attributed this effect to inner damping [3], which illustrates the close connection between rotordynamics and stability theory [4].

Active bearings enabled new possibilities to tackle rotor vibrations. In the 1990s, Palazzolo investigated active bearings based on piezoelectric actuators [5–7]. The introduction of active magnetic bearings in the 1980s led to an intense research of control algorithms. The cancellation of bearing forces caused by unbalances became an intensively discussed topic because they led to unwanted machine vibration on one hand, but also caused unnecessary amplifier utilization which

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degraded the bearing's performance on the other hand. Researchers developed many control algorithms to remove unbalance forces, for example Open-Loop Adaptive Control [8], Adaptive Autocentering Control [9], Adaptive Feedforward Compensation [10], Adaptive Forced Balancing [11], Automatic Inertial Autocentering [12], Periodic Learning Control [13], Unbalance Compensation [14] and Unbalance Force Rejection Control [15]. Despite their different mathematical description and justification, Herzog recognized that all approaches share the same notch-filter behavior [16].

Despite inarguable advances, the proposed algorithms have several shortcomings. First, many algorithms need a rotor model to perform in a stable manner [12–14] which makes their practical application tedious. Second, it is unclear how the different cancellation methods like Filtering or Feedforward compensation are linked together, leading to vague statements about the principle of operation [17,18]. The situation worsens as different active bearing types require different control algorithms, yet no one established a firm link for a generalized view. Furthermore, many approaches originate from a signal-theoretic viewpoint, often leading to an oversimplification of the underlying rotordynamics and a lack of physical insight. Additionally, many algorithms rely on complex mathematical concepts, making a thorough and general mathematical treatment of the system behavior impossible. The algorithm's effectiveness is usually proven using numeric simulation, having the drawback that the obtained results are valid for the specific system setup only. Finally, Larsonneur noticed that a reduction of bearing forces also leads to a reduction in rotor displacements and correctly identified the root cause [19], but could not give a mathematical description of the effect. In our previous work, we proved that unbalance forces and resonances can be eliminated both theoretically and practically on a Jeffcott rotor when a special collocated, model-free controller is used [20].

In this work, we broaden the approach to general rotors and found a *closed-form analytical solution for the elimination of unbalance-induced bearing forces on arbitrary, isotropic rotors*. We demonstrate that the force-free condition is met for any unbalance distribution. Furthermore, we proved that up to 2 resonances can be fully eliminated, giving the mathematical background to Larsonneur's observations. The appeal of our proof is not only in its completeness, but also in its comprehensible, graphical idea that does not require transfer functions. Our considerations are valid for arbitrary numbers, placements and types of active bearings.

Another highlight of our contribution is the stability analysis of the controlled system. Using Lyapunov's stability criterion, we discovered a *new stability theorem for mechanical systems with collocated controllers* and use it to show that our proposed controller is stable under almost any condition. Surprisingly, no rotor model is needed to guarantee stability. In addition, stability is also guaranteed for gyroscopic rotors. This favorable property is also known as *passivity* or *hyperstability* in literature [21,22]. We acknowledge that the presented stability proof does not cover systems with inner damping and circulatory forces.

As we demonstrated in our previous work [20], we establish a firm link between different active bearing principles and show that both force and displacement controlled actuators lead to the same system representation and can be covered by the same, generalized theory. We believe that the presented theory is not only useful in rotordynamics, but also extends to general systems with harmonic excitation that share similar matrix properties.

Finally, we test our theory on a test rig with a flexible rotor and an active bearing plane with piezoelectric actuators. Even though our theory treats all components as ideal, the experimental results match their theoretical counterpart very well. The results not only confirm the validity of our assumptions, but also demonstrate the effectiveness of our simple and model-free control algorithm for the elimination of unbalance forces.

2. Mechanical model

2.1. The free rotor

In this section, the properties of elastic shafts and their matrix representation are discussed. The limitation to isotropic rotors enables the use of the space-saving complex notation. We further assume that the rotor unbalance affects only the radial rotor motion, and consequently neglect axial rotor movements. The rotor's flexible properties are determined using the methods of matrix structural analysis, the most popular being the Finite Element Method [23,24]. All methods discretize a continuous structure into a finite number of flexible elements and store their properties in a stiffness matrix [25,26].

We assume that the shaft discretization leads to p degrees of freedom which are concatenated in the shaft displacement vector $\mathbf{q}_W = [q_{W1} \dots q_{Wp}]^T$. The stiffness properties of the *free* shaft are represented by the stiffness matrix $\mathbf{K}_R$ with the dimension $(p \times p)$. The relation between the shaft's node forces $\mathbf{F}_R$ and their corresponding displacements $\mathbf{q}_W$ is given by Hooke's law.

$$\mathbf{F}_R = \mathbf{K}_R \mathbf{q}_W \tag{1}$$

Although there are different methods to derive the free rotor's stiffness matrix $\mathbf{K}_R$, they result in the same matrix properties. First, it is symmetric due to Maxwell and Betti's reciprocal theorem [27]. Second, Fig. 1 indicates that there is a set of displacements that move or rotate the shaft as a whole but leave it unbent. These rigid body motions cause no node forces [28]. The stiffness matrix $\mathbf{K}_R$ maps shaft translations $\mathbf{q}_{Wt}$ as well as shaft rotations $\mathbf{q}_{Wr}$ or any linear combination of both to the null vector. A matrix property where both translations $\mathbf{0} = \mathbf{K}_R \mathbf{q}_{Wt}$ and rotations $\mathbf{0} = \mathbf{K}_R \mathbf{q}_{Wr}$ lead to vanishing forces is