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Extension of Hencky bar-net model for vibration analysis of rectangular plates with rectangular cutouts

Y.P. Zhang [a], C.M. Wang [b,*], D.M. Pedroso [b], H. Zhang [b]

[a] School of Mechanical and Mining Engineering, The University of Queensland, St Lucia, Queensland 4072, Australia
[b] School of Civil Engineering, The University of Queensland, St Lucia, Queensland 4072, Australia

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A B S T R A C T

Comprising rigid bars and spring systems at the joints and in the cells, the Hencky bar-net model (HBM) has been shown to be a physical structural representation of the finite difference plate model (FDM). In this paper, the HBM is extended for the vibration analysis of rectangular plates with rectangular cutouts. This extension addresses the rotationally elastic and transverse spring stiffnesses for the HBM at the cutout corners. After verifying the HBM model by comparing the vibration results with existing solutions for some plate problems, the model is used to obtain some new free vibration solutions for plates having various boundary conditions including cracked corners modelled by free small rectangular cutouts.

1. Introduction

Plates with cutouts (or perforated plates) are used in many structures and machines to provide access space or for weight reduction. The design of such plates requires performing free vibration analysis. There are many methods developed for the vibration analysis of plates with cutouts. They include the finite difference method [1–3], the finite element method [4–7], the boundary element method [8,9], and the differential quadrature element [10,11]. Some semi-analytical solutions for plates with cutouts were obtained by using the Rayleigh-Ritz method with domain decomposition method [12] and multi-term formulation of the extended Kantorovich method [13]. In this study, we develop the Hencky bar-net model (HBM) for free vibration analysis of thin isotropic rectangular plates with rectangular cutouts.

The Hencky bar-net model discretises the continuum plate by using rigid segmental bars and frictionless hinges with elastic rotational springs, twisting springs and transverse springs [14–17]. The HBM has been applied to solve beam [18–21], column [22,23], arch [24,25] and rectangular plate [26–28] problems. It should be noted that the HBM has also been called by different names such as the discrete element model [29–31], segmented rod/column [32,33], discrete link-spring model [34] and microstructured model [35]. The HBM has the following good features:

(i) the spring rotational stiffnesses at the joints of HBM can be easily modified to simulate local damage, local stiffening/ softening of plates.

* Corresponding author.
E-mail address: cm.wang@uq.edu.au (C.M. Wang).

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(ii) it is easy for computer coding with good convergence behaviour and versatile to accommodate different mixed boundary conditions, discrete loadings, and abrupt changes in plate properties.
(iii) it could be adopted for optimisation of non-uniform thickness plates against bending, vibration and buckling. For example, Zhang et al. [36] have used HBM to optimise the shape of columns under various boundary conditions because it allows the development of an efficient semi-analytical method that does not require resizing of elements and recomputing of stiffness properties during the optimisation process [37].
(iv) it could be applied in the alternating direction implicit method [38,39] to perform bending analysis of non-uniform thickness plates because of the reduction in the bandwidth for the global stiffness matrix [40,41].

Continuing in the development of HBM for plate analysis, we consider herein rectangular plates with cutouts. The inclusion of a rectangular cutout requires the introduction of a special rotational spring at the corners of the cutout. The finite difference method (FDM) will be used to aid in the determination of the rotational spring stiffness at the cutout corners. This work will enable the HBM to handle rectangular plates with rectangular cutouts as well as plates with cracks at the corners of the cutouts where high-stress concentrations occur.

The subsequent sections of the paper are organised as follows: Section 2 presents the problem definition. The governing equation and edge conditions for the free vibration of rectangular plates with a rectangular cutout are discussed in Section 3. Section 4 presents the HBM governing equation and the boundary conditions for the key nodes at the cutout corner from the energy approach. In Section 5, we derive the expressions for the stiffnesses of the rotationally elastic springs and transverse springs at the cutout corner by comparing the boundary conditions given by FDM (presented in the Appendix) and HBM. In Section 6, some plate free vibration plate problems involving rectangular cutout are solved, and special cases are compared to existing solutions to verify the HBM. Also in Section 6, new free vibration solutions for rectangular plates under various cutout boundary conditions are presented. Section 7 gives the concluding remarks. In the Appendix, we formulate the finite difference expressions for key nodes in the vicinity of the cutout corners.

2. Problem definition

Consider a rectangular plate with length αL, width L, cutout length ξL, cutout width ζL, uniform thickness h, mass density ρ, Young's modulus E and Poisson's ratio υ as shown in Fig. 1. The plate edges are restrained with elastic rotational springs having stiffness K_r^m/L and elastic transverse springs with stiffness K_l^m at all edges where m indicates the edges or corners. Similarly,

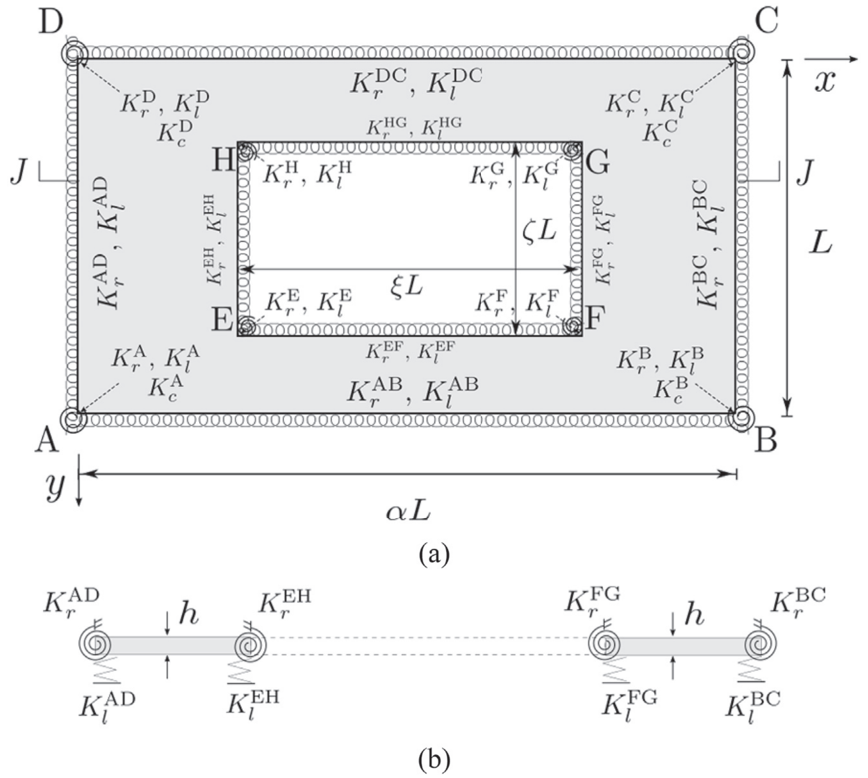

Fig. 1. Geometry of rectangular plates with elastically restrained edges and a rectangular cutout: (a) plan view; (b) view of section J-J.