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High Resolution Wavenumber Analysis (HRWA) for the mechanical characterisation of viscoelastic beams

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ABSTRACT

The High-Resolution Wavenumber Analysis (HRWA) is presented. It identifies complex wavenumbers and amplitudes of waves composing the harmonic response of a beam. Based on the frequency dependence of these wavenumbers, experimental dispersion equations of various beam mechanisms (e.g bending, torsion) can be retrieved. The HRWA method is compared to the *Mc Daniel* and the *Inverse Wave Correlation* (*IWC*) methods. It overcomes some drawbacks of these methods: the wavenumber resolution is enhanced. Also, the wavenumber search problem is expressed as a linear problem, making the method computationally efficient. A number of wavenumbers can be identified automatically, thanks to a statistical criterion. First, the noise sensitivity of each method is investigated in the basis of synthesised measurements. For this criterion, the HRWA and *Mc Daniel method* performances are close and much better than *IWC*. Moreover, the HRWA is five to twenty times faster to compute than other methods, depending on the mesh size. Second, an experimental case is presented where bending and torsion waves are identified, yielding an apparent viscoelastic Young and shear moduli on a wide-frequency range.

1. Introduction

With the apparition of full-field contactless measurement devices (e.g. scanning doppler laser vibrometers or high-speed cameras), displacement or velocity measurements can be performed on fine meshes, with a large number of points. Thanks to this major improvement, non-destructive structural characterisation methods have been developed [1–4]. Some of them focus on the identification of structural waves in the *medium frequency* domain [5–9]. These methods aim at filling the gap between low-frequency methods, (*Oberst* [10,11], modal analysis [12–16] or Dynamical Mechanical Analysis (DMA) [17]) and ultrasonic testing [18–20].

The present work focuses on applications to beam, considered as waveguides. Waves travelling in these structures are representative of the beam section motions: for example bending, twist or longitudinal motion. In these unidimensionnal structures, the wavenumbers are characteristic of the local structural behaviour: far from singularities, boundary conditions and sources have an influence on the wave amplitudes only. These wavenumbers are given by reduced beam models (e.g. Euler or Timoshenko models) or more elaborated schemes like Wave Finite elements [21]. The experimentally extracted wavenumbers can be used to identify apparent viscoelastic parameters of reduced models via an inverse problem [9].

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The existing Mc Daniel [7] and IWC [5,6] (Inhomogeneous Wave Correlation) methods are two candidate for the extraction of waves in the harmonic response of beams. However, these methods have some drawbacks (i) in low frequency, the Fourier based IWC method suffers from resolution limitations; (ii) in practice, the beam response is composed of multiple wave types (e.g. twist, compression, shear) and both existing methods fail to separate the different wave contributions, as they postulate the number of waves present in the signal; (iii) the formulation of these methods leads to a computationally expensive non-linear problem that has to be solved, with a complex wavenumber as parameter.

The aim of the High Resolution Wavenumber Analysis (HRWA) presented in this paper is to overcome the limitations of the existing wavenumber extraction techniques. It makes use of the subspace-based identification algorithm ESPRIT [22] (*Estimation of Signal Parameters via Rotational Invariance Techniques*). Subspace-based methods are widely-used in linear system identification, using for example the state-variable framework [23,24]. Another example is the ERA [25] (*Eigenvalue Realisation Algorithm*), which is devoted to the identification of the modal parameters of a measured system. Thanks to the use of the ESPRIT algorithm, some limitations of the Mc Daniel and IWC methods are overcome: (i) the algorithm resolution is high, as it uses a recurrence property of the signal to identify the wave parameters; (ii) by using the subspace decomposition, the number of waves contained in the signal can be estimated automatically with the ESTER criterion [26]; (iii) the complex wavenumbers are the solution of an optimisation-free problem thus the computational cost is lightened.

With the HRWA, a discrete number of complex wavenumbers is identified in the harmonic response of a beam with a high resolution. Based on the dependence of the extracted wavenumbers on frequency, experimental dispersion branches are retrieved. From these branches are identified the beam viscoelastic properties. Thanks to the high resolution aspects of the HRWA, the low-frequency limit of wavenumber extraction is lowered. Also, with the automated identification of multiple wavenumbers, strain mechanisms can be well separated, extending the upper frequency limit.

The paper is organised as follows. Firstly, theoretical wavenumbers are derived from the Euler and the Timoshenko beam models. The ability to identify beam properties from the wavenumbers is illustrated. Then, a common framework of wavenumber extraction methods is given, and the existing IWC method and McDaniel method reformulated in this framework. Secondly, the HRWA is developed in details. A summary of the algorithm is given, that allows to identify viscoelastic properties of the beam. Thirdly, a numerical study based on synthesised harmonic responses of a Euler beam in bending motion only is developed, where the Mc Daniel method is taken as reference. The Mc Daniel method, IWC method and the proposed HRWA are compared in terms of sensitivity to noise ratio and computation time. Finally, an experimental result is given on the simultaneous identification of frequency-dependent apparent viscoelastic Young and shear modulus, for both beam models.

2. Natural wavenumbers in a beam

All along the paper, the beam section is considered as homogeneous and made of a linear viscoelastic isotropic material with density ρ, Young modulus E and shear modulus G. However, the present method is applicable to higher-order beam models and more complex material configurations. The axis and the geometry of the beam are presented in Fig. 1 (x being the beam's direction). Not considering the bending motion along y, the beam neutral axis is assumed to remain in the (O, x, z) plane.

2.1. The Euler model

For the sake of simplicity, the Euler beam model is used for the numerical case considered in this paper. The linearised displacement field u is given by:

$$\boldsymbol{u}(x, y, z, t) = V(x, t)\,\boldsymbol{e}_x + W(x, t)\,\boldsymbol{e}_z - z\,W'(x, t)\,\boldsymbol{e}_x + \Theta(x, t)\left(y\,\boldsymbol{e}_z - z\,\boldsymbol{e}_z\right) \tag{1}$$

where V and W are respectively the longitudinal and transverse displacements, Θ is the rotation angle of the beam section with respect to the x axis, and $\bullet'$ denotes the partial derivative with respect to x.

Along the paper, the harmonic dependence on time of real physical quantities is accounted for by making them complex according to the $e^{\mathrm{i}\omega t}$ convention (where i denotes the imaginary unit). The local harmonic response of the beam at the angular frequency ω (except on sources or at boundaries locations) obeys the following uncoupled linear homogeneous equations:

$$E\,V'' = -\omega^2 \rho\, V \tag{2a}$$

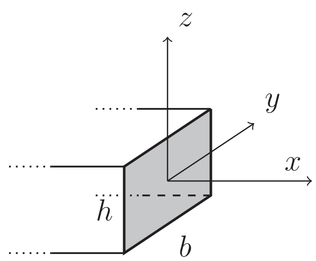

Fig. 1. Beam geometry and coordinate axis.