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Modal evaluation and generalized analysis of the steady-state dynamics of harmonically excited multistable structures

Benjamin A. Goodpaster [a], Ricardo A. Perez [b], Ryan L. Harne [a], *

[a] *Department of Mechanical and Aerospace Engineering, The Ohio State University, Columbus, OH, USA*
[b] *Universal Technology Corporation, Dayton, OH, USA*

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ABSTRACT

Predictions of multistable structural dynamics are paramount to the development and deployment of air vehicles operating under extreme loading conditions. Although time-stepping numerical techniques may capture the multi-physics interactions that occur in these environments, the generalized insight on parameters that predominantly govern the system behaviors may remain clouded while a large computational expense may be incurred to obtain response predictions. Alternatively, analytical methods may be employed to streamline the prediction process, yet current theoretical approaches do not facilitate such opportunity for multistable structures. Although a recently developed analytical formulation has enabled the prediction of near- and far-from-equilibrium responses for a simplified multistable structure, the preliminary formulation does not illuminate the underlying aspects of modal response and intricate nonlinear coupling manifest in myriad multistable systems. This research rectifies these limitations by a broad expansion of the analytical framework that empowers a new modal perspective of multistable structural dynamics and enables the study of such dynamic systems governed by reduced order models. This new modal analysis indicates that the characteristic frequency response of a single degree-of-freedom Duffing oscillator is preserved in the fundamental equivalent nonlinear mode of a multistable structure. The new analytical formulation is also shown to accurately predict the near- and far-from-equilibrium dynamics of equation systems containing global nonlinear coupling consistent with reduced order models. The advancements achieved in this work contribute to the suite of techniques available to researchers to characterize the near-to- and far-from equilibrium behaviors of nonlinear dynamic systems consisting of many degrees-of-freedom.

1. Introduction

From sub-orbit flight initiatives [1], to supersonic passenger jets [2], to hypersonic air vehicles [3]: applications abound for high-performance aerospace vehicles operating in extreme environments. Yet, the full realization of such air vehicles is impeded due to the combined thermo-mechanical-acoustical loading of high velocity flight regimes [4] that may stress slender aerostructural components into states of multistability, termed skin-buckling [5]. Once deformed, structural panels may exhibit a large-amplitude, snap-through dynamic response that can accelerate structural damage measures, lower

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* Corresponding author.
E-mail address: harne.3@osu.edu (R.L. Harne).

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fatigue life, and amplify the chances of catastrophic failure [6,7]. To surmount these challenges, recent works by Miller and McNamara [8] and Culler and McNamara [9,10] have advanced simulation of the complex fluid-thermal-structural coupling present for multistable panels in hypersonic flow. Miller et al. [11] gave particular attention to simulate the snap-through phenomenon in a multi-physics load environment. The computational demands encountered in such time-step simulation-based studies have provided motivation for the development of reduced order models (ROMs). ROMs decrease the dimension of the equation system thus lowering computational time, while prediction fidelity is balanced in the process. ROM-based characterizations of multistable structures have been reported by Wiebe and Spottswood [12], Mignolet et al. [13], and Matney et al. [14]. These numerical investigations have delivered new knowledge on the dynamic responses of multistable structures in extreme operating environments. Yet, the insight on such nonlinear structural behavior remains limited through the undertaking of case studies by simulation methods alone.

Analytical methods that approximately solve the governing equations of motion of nonlinear systems may greatly enhance the understanding revealed on the system response by way of meaningful simplifying assumptions [15]. Perturbation techniques can uncover the nonlinear, frequency-dependent displacement amplitudes of intrawell oscillations up to an arbitrary order of the small perturbation parameter ε [16]. Alternative forms of the standard perturbation technique, such as parameter-expanding methods [17,18] and homotopy perturbation [17,19], may be used to study any nonlinear dynamical equation [18]. Yet the utilization of higher orders of the expansion parameter ε and determining the appropriate parameter expansion challenges the use of perturbation techniques for complex systems of coupled nonlinear equations [20]. In fact, many analytical investigations on the topic of multistable structural dynamics have given attention to bistable systems due to the relative simplicity of the theoretical formulations by virtue of the single degree-of-freedom (DOF). The narrower attention to bistable structures is also encouraged due to the wide range of applications from energy harvesting [21,22], to vibration control [23], to sensing [24]. The relative simplicity of the theoretical formulations for such single-DOF systems makes it possible to derive closed-from predictions of the strongly nonlinear responses of bistable oscillators [24].

Such straightforward analytical solution is often not possible for larger dimensional nonlinear systems [24]. As a result, a variety of methods have been employed to study multi-DOF nonlinear systems. Ritz methods utilizing quadratic polynomials of strain and deflection were shown by Mattioni et al. [25] to characterize the nonlinear deflections of multistable composite plates. Pirrera et al. [26] demonstrated that higher-order polynomials may be used to describe multi-event snap-through phenomena for such multistable composite structures. Describing function theory, propelled by Tanrikulu et al. [27], Chong and Imregun [28], Elizalde and Imregun [29], and Kalaycıoğlu and Özgüven [30], has delivered understanding on the steady-state, forced response that exacerbates the nonlinear resonant displacements of multi-DOF structures. While the formulations may require the system to exhibit symmetric nonlinearities [27] or may require an assumption of well-separated modes of vibration [28,31], a notable limitation of these methods is the assumption of input-output similarity. This prevents one from characterizing the co-existence of large amplitude snap-through dynamics and low amplitude oscillation around non-zero equilibria that may occur for multistable structures [11]. Alternatively, principles of equivalent linearization [32] have been used to characterize such near-to- and far-from-equilibrium responses of structures subjected to stochastic or harmonic excitations [33–35]. These methods rely on the formation and convergence of an equivalent linear stiffness matrix that enables the linearized equations to be solved via more conventional methods of analysis. To this end, Harne and Goodpaster [36] recently introduced an experimentally validated analytical formulation based on harmonic linearization [32] capable of predicting the steady-state dynamics of a built-up multistable structure. In contrast to methods based on describing function theory, both near-to- and far-from-equilibrium dynamic behaviors are able to be uncovered. The formulation [36] is also distinct from other equivalent linearization techniques by virtue of the implicit calculation of equivalent stiffness matrix terms in the process of obtaining the coupled, nonlinear algebraic equations to be solved for system response prediction. On the other hand, the nascent formulation lacks the means to shed insight on modal characteristics of multistable structures that may be contrasted to those of traditional linear dynamic systems. In addition, the extensibility of the formulation to characterize nonlinear systems represented by conventional ROM equations [13] has not been established. As a result, the knowledge on multistable structural dynamics manifest in myriad nonlinear systems, and their contrast to established linear systems, remains lacking.

This research aims to rectify these limitations via a comprehensive research undertaking. First, a new construct of the analysis [36] solution algorithms is fashioned, which becomes necessary to facilitate the new complement to conventional modal analysis [37]. Then, this unique modal formulation is leveraged to shed light on steady-state dynamics of multistable structures as they contrast with those modal behaviors of linear systems. Next, the analysis is significantly broadened to provision the study of multistable structures whose dynamic behaviors are governed by ROM-type equation systems. With these advancements, the clarity of the predictions is rigorously compared with those results obtained through the traditional time-stepping based simulations.

The remainder of this paper is organized as follows. In the next section, the preliminary analytical formulation is summarized. Then, the necessary advancements to the formulation established here are detailed. These improvements are then utilized to study qualitative characteristics of far-from-equilibrium dynamics in a modal perspective. The framework is then extended to accommodate nonlinearities and structural coupling manifest in ROM-based multistable structure models, and the efficacy and efficiency of the analytical predictions are characterized. A final section summarizes the achievements made and new understanding created through this research.