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High order asymptotic dynamics of a nonlinearly coupled electromechanical system

Valeria Settimi *, Francesco Romeo

Department of Structural and Geotechnical Engineering, Sapienza University of Rome, 00197 Rome, Italy

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ABSTRACT

A nonlinearly coupled mathematical model of an electro-magneto-mechanical system is studied via the multiple scale approach in order to investigate its weakly nonlinear dynamics and analytically predict its salient features. The obtained amplitude modulation equations up to the third order perturbation allow to analytically describe the mechanical and electrical responses in terms of frequency-response curves and stability scenarios. A critical threshold of Hopf bifurcation is detected and analyzed as a function of the main system parameters. The subsequent extension of the asymptotic scheme up to the fifth order proves to grasp also the post-critical behavior, providing with the accurate identification of the amplitude of the quasiperiodic responses characterizing the unstable region of the ordinary differential equations system.

1. Introduction

The nonlinear dynamics of an electromechanical system is considered in this study. The system consists of a linear oscillator nonlinearly coupled through an electromagnet to a linear electric circuit. As it is known, electromechanical systems are characterized by the interaction among inertial, electric, and magnetic circuits, and they are nowadays widely used for devices that monitor and control machines and structural systems. Depending on the specific application, electromechanical devices range from macro to micro and nano scales. Irrespective of the scale, they can be modeled by lumped physical elements, such as mass or inductance, for which basic dynamic principles apply, leading to coupled ordinary differential equations (ODEs) [1]. The conversion between electrical and mechanical energy may take place in electromechanical motion devices through energy transfer due to coupling electromagnetic field.

In electromagnetic circuits, the dynamical equations take the form of conservation of charge and the Faraday–Henry law of flux change. If the system has a mechanical state variable, such as displacement, then the inductance may be a function of it. In such a case the magnetomotive force, induced by a current exciting a coil, acting on a ferromagnetic movable member depends on its displacement thereby triggering oscillations. By exploiting this principle, a number of devices have been proposed so far.

Magnetic solenoid actuators are electromagnetic devices that convert electrical energy into a mechanical push/pull force or motion finding a number of applications across different industries. Solenoids are variable reluctance translational machines integrating movable (plunger) and stationary (fixed) members made from high permeability ferromagnetic materials; in these actuators the inductance and the reluctance force generated both vary nonlinearly with position [2–4]. Synchronous reluctance motors are rotational electromechanical devices in which the magnetizing inductance is a function of the rotor angular dis-

* Corresponding author.
E-mail addresses: valeria.settimi@uniroma1.it (V. Settimi), francesco.romeo@uniroma1.it (F. Romeo).

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placement [3]. From a modeling perspective, in electrodynamic loudspeakers it has been recently proved that the nonlinearity of inductance as function of displacement is a significant source of distortion; a linear approximation of this voice-coil inductance dependence was considered in Ref. [5].

The nonlinear dynamics stemming from the electromagnetic coupling characterized by displacement dependent induction was also addressed by a number of studies related to classical problems, such as the Bethenod's pendulum, investigated, e.g., in Ref. [6]. In this system a pendulum with a ferromagnetic bob experiences growing and sustained oscillations when it is made to vary the inductance of a coil excited by high-frequency current. In this work, a quadratic law for the inductance dependence on the pendulum oscillation angle was used to approximate the experimental bell shaped profile. A similar electromechanical system was considered in Ref. [7]. The electromechanical system consists of a linear electrical circuit whose inductance varies linearly with the position of a bar. The bar, which is made of magnetic material forming part of the inductance core, represents the mass of a damped, linear, mechanical oscillator and is forced by the ponderomotive action of the electromagnetic field of the inductance. The nonlinear amplitude frequency response, merely of the electrical circuit, and a region of instability were analytically obtained using a modified Ritz averaging technique. Later, the same two degrees of freedom system was considered in Refs. [8,9] as an example of self-oscillatory system. Combination frequencies arise due to the nonlinear interaction between the dynamic variables, so that the oscillations become quasiperiodic. As a result of the interaction of these oscillations with the harmonic source, energy transfer from the high-frequency (electric) source to the low-frequency (mechanical) oscillations occurs. An asymptotic approach assuming a quasiperiodic electrical response lead to obtain the slowly varying functions describing amplitude and phase of the mechanical oscillator.

From the analytical point of view it is worth mentioning the similarity between the nonlinearly coupled behavior in liquid-shell systems and the electromechanical system at hand. The analogy in the governing differential equations is obtained by adopting a simplified liquid-shell system in which only one liquid mode is considered [7,10].

In Refs. [11–13] the same electromechanical system here tackled is numerically thoroughly investigated. The occurrence of significant bifurcations marking the transition from regular to non regular responses are detected. Moreover, a wide unstable region with high-amplitude quasiperiodic and chaotic responses characterizes the system's nonlinear dynamics. From a phenomenological viewpoint, it consists in inducing a qualitative and quantitative change on the mechanical oscillations, moving the system response from period-1 low amplitude oscillations to vibrations characterized by sensibly higher amplitude and very rich frequency spectrum. As a comparison, such scenario can be related to the well-known pull-in phenomenon, characterizing the dynamical response of several microelectromechanical/nanoelectromechanical (MEMS/NEMS) devices [14]. In both cases, in fact, they result in the sudden transition from low to high amplitude oscillations. However, depending on the device operation, it can represent an undesirable or a beneficial condition. While for instruments like micro-motors, capacitive sensors, radio frequency (RF) switches and magnetostatic actuators the unexpected jump to high amplitude oscillations leads to failure, the same phenomenon can also be used as the sensor mechanism, and create actuators that can generate a large amount of force, with the main advantage, with respect to the pull-in behavior, of working in a monostable, thus more robust, as concerns the dynamical response to be pursued, regime.

Aiming at predicting and describing this peculiar behavior while completing the outcomes presented in a companion paper [13], a purely analytical asymptotic approach is here proposed. This analysis turns out to be effective due to the persistence of the unstable region also for rather low values of the nonlinear coupling. In developing the asymptotic procedure, useful suggestions come from Ref. [11], where the analytical approach performed though a slow-fast time decomposition analysis needed to reach high orders to catch the dynamical phenomena. In the light of this, a high order Multiple Scale Method [15–17] is applied to the model. The paper is organized as follows. After the introductory section, and a brief description of the physical model under analysis in Section 2, the multiple scale formulation is presented in Sections 3 and 4; it is shown that the asymptotic approach up to the third order enables to identify the frequency response curve around the primary resonance and to analyze its stability. By extending the asymptotic approach up to the fifth order, the post-critical regime is then addressed in Section 5; the concluding remarks are eventually reported.

2. Modeling

The physical model herein considered is composed of a linear oscillator nonlinearly coupled to a linear electric circuit by means of an electromagnet, as shown in Fig. 1, which represents an archetypal model for different physical devices [9]. By referring to the general expression of magnetic inductance for elementary electromagnets $L(x) = k/(k_0 - x)$, with k and k_0 depending on the number of winding turns and on the material reluctance, the linearized form is here considered, i.e. $L(x) = L_0 + L_1 x$, highlighting the linear dependence of the inductance L on the mechanical displacement x. The equations of motion for the electro-mechanical system are:

$$\ddot{x}(t) + 2\zeta_m\omega_m\dot{x}(t) + \omega_m^2 x(t) = \frac{\xi\alpha\omega_m^4\dot{q}(t)^2}{\omega_e^4} \quad (1 + \alpha x(t))\ddot{q}(t) + (2\zeta_e\omega_e + \alpha\dot{x}(t))\dot{q}(t) + \omega_e^2 q(t) = \beta\omega_e^2\sin(\Omega t) \tag{1}$$

where $x(t)$ and $q(t)$ are the mechanical and electrical (charge) displacements, respectively, and

$$\omega_m^2 = \frac{k}{m},\ \omega_e^2 = \frac{1}{L_0 C},\ \zeta_m = \frac{c}{2\sqrt{km}},\ \zeta_e = \frac{R\sqrt{C}}{2\sqrt{L_0}},\ \xi = \frac{L_0}{2m},\ \alpha = \frac{L_1}{L_0}$$