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Eigenvalue and eigenmode synthesis in elastically coupled subsystems

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ABSTRACT

A method to synthesize the modal characteristics of a system from the modal characteristics of its subsystems is proposed. The interest is focused on those systems with elastic links between the parts which is the main feature of the proposed method. An algebraic proof is provided for the case of arbitrary number of connections. The solution is a system of equations with a reduced number of degrees of freedom that correspond to the number of elastic links between the subsystems. In addition the method is also interpreted from a physical point of view (equilibrium of the interaction forces). An application to plates linked by means of springs shows how the global eigenfrequencies and eigenmodes are properly computed by means of the subsystems eigenfrequencies and eigenmodes.

1. Introduction

Many products consist of an active device; e.g. engines, compressor units, frequency converters, etc. and a structure which is defined to meet a functionality target. Such structures may be for example trains, cars, refrigerators, etc. and the corresponding active devices which supply energy to the entire system are frequently categorized as auxiliary equipment.

Other type of products which may be partitioned in a similar way are all kind of machines installed on top of a supporting structure, as typically encountered in industrial facilities.

The assembly of the two aforementioned elements, typically through elastic joints (see Fig. 1), may generate noise and/or vibration complications due to the dynamic coupling between them. For instance, let an engine, whose dynamics are known, having no resonances at its fundamental excitation frequency. The engine is installed by means of elastic supports on a structure also not having resonances at excitation frequency. Then, the question that arises is whether the engine-structure assembly will resonate. A resonance of the assembly at the excitation frequency would be critical since it may cause noise issues as well as ill-functioning of the product.

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Symbol list

$\mathbf{A}$	Dynamic matrix of the mechanical system
$\mathbf{B}$	Diagonal block matrix involving matrices $\mathbf{B}_1$ y $\mathbf{B}_2$
$\mathbf{C}$	Coupling matrix, non-zero valued only at antisymmetric positions connecting $\mathbf{B}_1$ and $\mathbf{B}_2$ coefficients
$c_{P_iQ_i}$	Matrix $\mathbf{C}$ coefficient linking degrees of freedom labelled as P_i and Q_i
$c_{P_iQ_i}$	Matrix $\mathbf{C}$ coefficient linking degrees of freedom labelled as P_i and Q_i
$\mathbf{D}_B$	Diagonal matrix of eigenvalues of $\mathbf{B}$
$\mathbf{e}_{P_k}$	Base vector, unity-valued at degree of freedom labelled P_k
$\mathbf{e}_{Q_k}$	Base vector, unity-valued at degree of freedom labelled Q_k
$\mathbf{H}$	$\mathbf{B} + \mathbf{C}$
$\mathbf{K}$	Stiffness matrix
$\mathbf{M}$	Mass matrix, diagonal an positive definite
m	Number of degrees of freedom of $\mathbf{B}$ connected by $\mathbf{C}$
m_j	jj coefficient of the mass matrix
n_ϵ	Dimension of a generic matrix $\boldsymbol{\epsilon}$
P_i	Label for the i'th connection on subsystem 1 (may not correspond to degree of freedom i)
Q_i	Label for the i'th connection on subsystem 2 (may not correspond to degree of freedom i)
$\mathbf{u}_{P_k}$	Column vector in $\mathbf{V}_B^{-1}$ on degree of freedom labelled P_k
$\mathbf{u}_{Q_k}$	Column vector in $\mathbf{V}_B^{-1}$ on degree of freedom labelled Q_k
$\mathbf{V}_B$	Matrix of eigenvectors of $\mathbf{B}$
$\mathbf{v}_{P_k}^T$	Row vector in $\mathbf{V}_B$ on degree of freedom labelled P_k
$\mathbf{v}_{Q_k}^T$	Row vector in $\mathbf{V}_B$ on degree of freedom labelled Q_k
Γ	Characteristic function whose zeroes are eigenvalues of $\mathbf{H}$
λ_A	Any eigenvalue of matrix $\mathbf{A}$
$\lambda_{B_1,k}$	k'th eigenvalue of matrix $\mathbf{B}_1$
$\lambda_{B_2,k}$	k'th eigenvalue of matrix $\mathbf{B}_2$
$\mathbf{\Phi}_i$	i'th eigenmode of subsystem 1
$\phi_i(P_j)$	Coefficient in $\mathbf{\Phi}_i$ corresponding to degree of freedom P_j
μ_i	Modal mass of eigenmode i in subsystem 1, $\mathbf{\Phi}_i$
$\mathbf{\Psi}_i$	i'th eigenmode of subsystem 2
$\psi_i(Q_j)$	Coefficient in $\mathbf{\Psi}_i$ corresponding to degree of freedom Q_j
σ_i	Modal mass of eigenmode i in subsystem 2, $\mathbf{\Psi}_i$
ω	Angular frequency

Now, consider that the modal frequencies and mode shapes of each subsystem are known when the subsystems are subject to an infinite impedance condition through the elastic joints at their interface, c.f. Fig. 2. Then, the problem to be solved is determining the assembled system modal frequencies.

The problem of devising the behavior of a system from which one knows that of its two composing subsystems has been widely studied. The fields in which this problem has been addressed are widespread. As a matter of example one can cite in the field of electrical networks a method developed in Ref. [1], the method is known as the Kron method. It was immediately applied to dynamics in Refs. [2,3] and has been further developed recently in Refs. [4–7]. One can also cite methods in the field of physics of composite systems such as those applied to sets of phonons [8] or to composite materials [9]. Eventually, one may refer to methods in the field of dynamics, for which a non-exhaustive review of methods may be found in Ref. [10].

The latter reference exposes an historic review of methods from which it summarizes the problem in just three equations and, subsequently, it examines the different approaches for its solution in the physical space, the modal space or the frequency space. The first equation is the dynamic equation for each of the subsystems subjected to external forces as well as coupling reaction forces. The second equation defines a linear dependency between coupling interface coordinates, usually trough a boolean matrix. Finally, the third equation defines the relation between the forces that each subsystem exerts on the other.

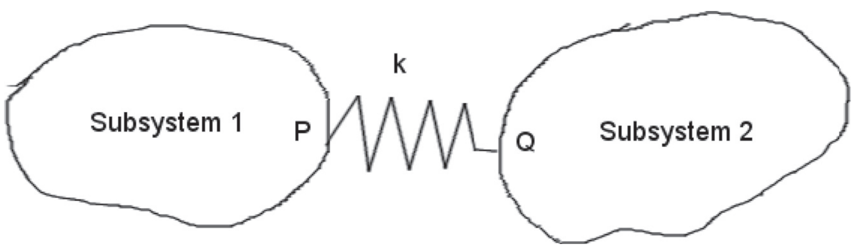

Fig. 1. Subsystems 1 and 2 with one or more coupling spring.