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Vibration analysis of a rotating pre-twisted blade considering the coupling effects of stretching, bending, and torsion

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ABSTRACT

An enhanced dynamic model for a rotating pre-twisted blade with arbitrary cross-section is proposed in this study. The coupling effects between stretching, bending, and torsion are considered simultaneously in this model. To derive the equations of motion, deformation variables, including a special variable representing stretch, are approximated using the assumed mode method. The stretch variable plays a key role in the coupling effects between stretching, bending, and torsion. The accuracy of the proposed model was validated by comparing the analysis results to those obtained using a commercial finite element code. The effects of several parameters, such as cross-section eccentricity, pre-twist angle, torsion to bending stiffness ratio, and angular speed, on the modal characteristics of the blade were investigated using the proposed model. It was found that coupled modal characteristics of a rotating pre-twisted blade could be accurately estimated with the proposed model.

1. Introduction

It is well known that the vibration characteristics of rotating blades differ substantially from those of non-rotating blades because rotational motion induces a centrifugal inertia force, resulting in stiffness variation in the system. The importance of stiffness variation has been long recognized in steam and gas turbine blades, turbo fans, and aircraft wings that undergo rotational motion. To avoid malfunction or premature fatigue failure of such a system, the vibration characteristics of rotating blades need to be estimated accurately. For this analysis, an accurate dynamic model is required.

In structural analyses, blades are often idealized as pre-twisted cantilever beams with asymmetric airfoil-like cross-sections. The centroid and elastic center of the blade cross-section generally do not agree and it is often called the eccentricity. The combination of the eccentricity and gyroscopic effect, which induces coupling between the bending and torsional motions of the rotating blade, causes blades to exhibit coupled motion between bending and torsion. Due to the pre-twisted shape along the longitudinal axis of the blade, the chordwise and flapwise bending motions are also coupled. Moreover, stretching induced by the centrifugal inertia force results in bending and torsional stiffness variation; therefore, stretching is also coupled with bending and torsion. For this reason, the coupling effects between stretching, bending, and torsion should be considered simultaneously to accurately investigate the vibration characteristics of a rotating pre-twisted blade with asymmetric cross-section.

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A large number of research papers on the subject of rotating blades have been published so far. Early pioneering research on the subject was conducted by Southwell and Gough [1]. Based on the Rayleigh energy theorem, they derived a simple algebraic equation, called the Southwell equation, to obtain the natural frequencies of a rotating blade. Later, partial differential equations governing the bending vibration of rotating blades were derived by Schilhansl [2]. Mode shapes as well as natural frequencies of rotating blades could be obtained from the equations using the Ritz method. As computers became popular and more widely available for researchers, various kinds of rotating blade vibration analyses based on numerical methods could be performed. Informative early literature survey papers have been published on this subject [3–5]. Accurate linear dynamic models for rotating beams were also introduced to obtain transient vibration analysis results [6,7]. Most of the results, however, were related to the bending vibration of rotating blades because the fundamental natural frequency of a blade generally corresponds to the bending mode. The overall dynamics involving in stretching, bending and torsion of rotating blades were rarely studied in a rectangular cross-section case [8,9]. The effect of coupling between bending and torsion on the vibration characteristics of a rotating blade was firstly investigated by Subrahmanyam et al. [10]. In that work, the effect of cross-section eccentricity on the elastic coupling between bending and torsion was considered. Another work done by Ozgumus and Kaya [11] included non-uniform cross-section effects on the coupled flapwise bending and torsional vibration of rotating Timoshenko beams using differential transform method. Avramov et al. [12] derived the equations of bending-bending-torsion coupled nonlinear vibrations of asymmetric blades. Coupled bending-bending-torsion vibration analyses were conducted for rotating pre-twisted blades using an integral formulation by Surace et al. [13]. Some works [14–16] were extended to thin-walled composite beams taking into account the effects of anisotropy and fiber orientation. Contrary to the previous works that the modeling methods are based on the beam theory, Sinha and Turner [17] applied the thin shell theory to consider the bending-torsion coupling effect of a rotating pretwisted model having a rectangular cross-section. On those papers, however, the effect of stretching on torsional stiffness variation and the gyroscopic effect were not properly considered. The gyroscopic effect was investigated by Bhadbhade et al. [18] and Cooley et al. [19] for vibrating beam gyroscopes. They showed that the bending and torsional motions of a blade could be coupled through the gyroscopic effect even when the blade had no cross-sectional eccentricity.

Wagner [20] and Biot [21] showed that the torsional stiffness of a blade could be varied using a compressive or extensional force. The effect of stretching on the torsional natural frequencies of a rotating rod was investigated by Yoo et al. [22] using a set of deformation variables that included a stretch variable. They were able to obtain accurate torsional natural frequencies of a rotating rod; however, the simultaneous effects of coupling between stretching, bending, and torsion were not considered in any previous works.

The main novelty of the present work is the simultaneous consideration of the coupling effects between stretching, bending, and torsion that occur in a rotating pre-twisted blade with an asymmetric cross-section. The proposed dynamic model consists of linear equations of motion that can be conveniently used for transient analysis as well as modal analysis of a rotating pre-twisted blade with an asymmetric airfoil-like cross-section. To verify the accuracy of the proposed modeling method, the natural frequencies of rotating pre-twisted blades obtained using the proposed modeling method were compared to those obtained using a commercial finite element code [23]. To overcome the difficulty of representing 3-dimensional mode shapes of a blade, a novel method using colors and their intensity was introduced in this study. With the proposed method, mode shape variations as a function of the angular speed of blade could be effectively exhibited.

2. Dynamic modeling of the vibration analysis

2.1. System configuration

Fig. 1 shows the undeformed and deformed shapes of a pre-twisted blade with an asymmetric cross section. The blade is modeled as a Timoshenko beam with natural length L. It is fixed to a rigid shaft S of radius r, which rotates with a constant

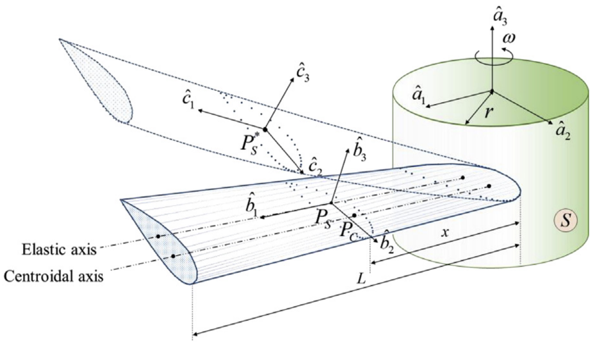

Fig. 1. Undeformed and deformed shapes of a pre-twisted blade fixed to a rotating rigid shaft.