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A wave finite element approach for the analysis of periodic structures with cyclic symmetry in dynamic substructuring

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ABSTRACT

A numerical approach is proposed to compute the dynamic response of periodic structures with cyclic symmetry, and assemblies made up of these periodic structures. The wave finite element (WFE) method is used to describe the wave modes which occur around the circumferential direction of these periodic structures. Emphasis is placed on assessing the dynamic flexibility modes of a periodic structure by considering unit forces which are successively applied to the boundary degrees of freedom. It is shown that the matrices of dynamic flexibility modes can be quickly computed. This yields an efficient dynamic substructuring technique to analyze the dynamic behavior of assemblies made up of several periodic structures. Numerical experiments are carried out which concern the analysis of a single periodic structure as well as assemblies made up of two and three structures.

1. Introduction

A numerical approach is proposed to compute the dynamic response of periodic structures with cyclic symmetry. Such systems are frequently encountered in the aeronautic industry in the design of turbines. Predicting their dynamic behavior by means of accurate and efficient numerical tools has become a key challenge which is motivated by the need of considering systems of increasing complexity — i.e., systems which involve finite element (FE) models with many degrees of freedom (DOFs). To address this issue, the FE method [1] and the theory of cyclic symmetry [2] can be used. However, both of these approaches suffer from strong issues. These mainly concern the numerical cost involved when assembling and inverting large-sized sparse matrices in an FE model, and, concerning the theory of cyclic symmetry, the use of an assumption consisting of neglecting the coupling between the harmonic modes (nodal diameters) of two connected periodic structures [3]. As a result, the CPU times involved in the FE method are likely to be excessive, while the accuracy of the theory of cyclic symmetry cannot be guaranteed when coupled systems are dealt with. The present paper aims at proposing an alternative approach to circumvent these drawbacks.

A new numerical approach is proposed which involves considering the wave finite element (WFE) method. In this framework, the so-called wave modes of a periodic structure with cyclic symmetry are computed. These are to be understood as the waves which propagate/travel around the circumferential direction of the structure. The wave modes are further used to express the dynamic flexibility modes of the structure, which is done by analyzing its dynamic response when unit forces are applied to the boundary DOFs. The interesting and original feature of the proposed approach is that the matrices of dynamic flexibility modes can be quickly computed, leading the way to an efficient dynamic substructuring technique to analyze assemblies made up of

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several periodic structures.

Basically, the WFE method can be viewed as a transfer matrix approach to describe the wave propagation along one-dimensional periodic structures, i.e., structures composed of identical substructures which are connected to each other along a straight or circumferential direction. So far, the WFE method has been applied to different kinds of homogeneous and heterogeneous structures [4–9], elasto-acoustic systems [10,11], poroelastic media [12], and so on. The WFE method works by considering the FE model of a substructure — i.e., a periodicity pattern which can have arbitrary 2D or 3D shape — which can be obtained using a commercial FE software. Hence, the dynamic stiffness matrix (DSM) of the substructure can be expressed (from the known mass, damping and stiffness matrices) which is further condensed on the interface DOFs, that is, those on the coupling interfaces with the previous and subsequent substructures. By considering the condensed DSM, the transfer matrix of the substructure can be expressed. It links the displacement and force vectors on one interface of the substructure (say, the right boundary) to those on the second one (left boundary). The wave modes of a periodic structure are finally obtained by computing the eigenvalues and eigenvectors of the transfer matrix of the substructure. These are referred to as the wave parameters (wave numbers) and the wave mode shapes, respectively.

Also, the WFE method has been applied to compute the harmonic forced response of periodic structures [13–17]. In this framework, finite-length periodic structures — i.e., structures with a finite number of substructures — are dealt with, whose ends can be subject to different kinds of boundary or coupling conditions. In this case, assumption is made that the DOFs which do not belong to the end boundaries of the periodic structures are free from excitation sources. The WFE strategy consists in expressing the displacement and force vectors of a periodic structure, at the substructure interfaces, in terms of wave mode shapes. This yields small-sized wave-based matrix equations which can be solved efficiently. The analysis of assemblies made up of periodic structures which are coupled together (on their end boundaries), or which are coupled to elastic junctions, can be undertaken in the same way and requires the coupling conditions to be expressed in wave-based forms. The interesting feature of the WFE method is that it is faster than the FE method, while keeping the same level of accuracy.

In addition, it should be emphasized that the WFE method has been applied to the analysis of two-dimensional periodic structures like bi-periodic plates built from a 2D array of identical substructures [18]. This approach has been mostly applied to identify Bloch waves in 2D periodic structures, and analyze the related band gap effects, i.e., frequency regions on which Bloch waves do not propagate. The counterpart of this approach, however, is that it is restricted to the analysis of the 2D wave propagation in infinite periodic structures. The analysis of bounded structures, e.g., with a surrounding boundary on which boundary conditions are considered, is an extremely tough issue which has never been carried out so far from the author's point of view. The scientific challenge mostly relies on the modeling of the reflection and transmission phenomena of Bloch waves, which are multi-directive, on the boundary of the periodic structures.

The underlying question behind the WFE method, for one-dimensional periodic structures, may be stated as to how to widen its range of application so that it can be applied to real engineering problems. In Ref. [19], it has been shown that the WFE method is capable of handling periodic structures made up of arbitrary shaped substructures whose FE models can contain many DOFs (e.g., more than 10, 000). One main limitation of the WFE method, however, is that the excitation sources are considered at the structure ends, which especially means that the substructures are supposed to be free from excitations. The present work aims at tackling this problem within the specific scope of periodic structures with cyclic symmetry, e.g., with an inner circumferential boundary subject to a distributed excitation. This leads the way to the analysis of coupled problems involving several periodic structures with cyclic symmetry which are connected together around their boundaries. In this sense, a dynamic substructuring technique can be proposed in the framework of which several periodic structures are modeled in terms of dynamic flexibility modes. The analysis of assemblies made up of several periodic structures follows from a domain decomposition procedure by enforcing displacement continuity conditions at coupling DOFs.

The scientific novelties of the present work can be summarized as follows:

- Modeling of periodic structures with cyclic symmetry whose inner circumferential boundary is subject to a distributed excitation.
- Numerical approach for quickly computing the matrices of dynamic flexibility modes of periodic structures, and modeling assemblies made up of several periodic structures which are connected together across their inner circumferential boundaries.

The rest of the paper is organized as follows. In Section 2.1, the FE model of a substructure which is used in the WFE method is presented. The WFE strategy for computing the wave modes of periodic structures with cyclic symmetry is presented in Section 2.2. Also, the WFE strategy for computing the forced response of periodic structures with cyclic symmetry is detailed in Section 2.3. In Section 2.4, the derivation of the matrices of dynamic flexibility modes is proposed. The analysis of assemblies made up of several periodic structures is proposed in Section 2.5. Finally, numerical experiments are brought in Section 3. These concern the study of one single periodic structure (Section 3.1), and assemblies made up of two and three periodic structures (Sections 3.2 and 3.3). The relevance of the WFE approach is analyzed through comparisons with the FE method and the theory of cyclic symmetry.