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Wave propagation characteristics of periodic structures accounting for the effect of their higher order inner material kinematics

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A B S T R A C T

The current work investigates the wave dispersion characteristics of two-dimensional structures, accounting for the effect of their higher order inner material kinematics. For the computation of the nonlinear dispersion diagram, a perturbation approach appropriate for the incorporation of nonlinear effects on the linear band structure attributes is employed. The method is used to compute the dispersion characteristics of architectured periodic materials, structured with hexagonal, re-entrant hexagonal, as well as square and triangular-shaped unit-cells. The corrected nonlinear dispersion characteristics suggest that the incorporation of the higher order kinematics induced corrections, entail a wave amplitude and wavenumber dependent mechanical response. Furthermore, the numerical simulations demonstrate that nonlinear effects primarily arise for the lowest rather than for the higher eigenmodes. What is more, it is shown that the highest magnitude corrections are expected for the lowest shear mode in the low frequency region, while for a given wave amplitude, the unit-cell design plays a significant role in the magnitude of the obtained nonlinear correction.

1. Introduction

The wave propagation characteristics of periodic network structures are determined from the geometric and material attributes of their inner architecture [1,2]. Up to now, considerable research efforts have been made to study the propagation of elastic waves in two-dimensional media [3,4]. The inner material architecture has been commonly optimized to design topologies that allow for the creation of band-gaps, thus frequency regions in which no waves propagate [5]. In the case of material architectures with a repetitive structural pattern for which a unit-cell can be identified, the Bloch's theorem has been commonly employed for the computation of the material's wave propagation attributes [6]; the consideration of periodic network or architectured materials entails invariance by spatial translation. This has the far-reaching consequence in terms of the so-called Bloch wave conditions and the analysis of wave propagation in an infinite periodic structure can be reduced to

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the analysis of wave propagation in a single cell representative of the entire structure, associated to the Bloch wave conditions. Typical examples of the case constitute hexagonal and auxetic unit-cell shaped metamaterials [7].

However, the occurrence of non-linearities can considerably affect the wave propagation characteristics of structures, with their modeling to constitute a substantial challenge even in the one-dimensional case [8]. Finite strains require a non-linear analysis process for an accurate characterization of their wave attributes [9–11]. The arising structural nonlinearities can be primarily classified to two major categories: material [12,13] and geometric nonlinearities [14]; network materials with slender structural elements are prone to geometrical nonlinearities (due e.g. to the low flexural rigidity of the structural members), with local strains still remaining small so that materials nonlinearities can be excluded.

The dynamic equilibrium equations which characterize the wave propagation characteristics of non-linear structural systems support a variety of solutions, depending on the type of nonlinearity [15]. A series of new phenomena not observed for linear systems appear. In particular, the dispersion characteristics of structures become wave amplitude dependent, while different wave types can appear, such as solitary waves and localized out-of-phase standing waves, which exhibit an intrinsic, non-linear dynamic behavior [16,18]. The dependence of the dispersion relation diagram on the amplitude of the propagating waves has opened new possibilities in the so-called passive tuning of the dispersion band structure, thereby going beyond a mere control of the dynamic and acoustic structural properties by their initial design [17].

In cases of geometric nonlinearities, the wave attributes of structures are directly affected by the application of initial strains. The initial deformations applied must be large enough to change the domain geometry, since infinitesimal strains would not substantially modify the inner material structure [19]. It is worth mentioning that most of the studies of nonlinear wave propagation deal with one-dimensional or two-dimensional systems [20,21] under the effect of pre-stressing, whereas non-linear wave propagation accounting for geometric nonlinearities in three-dimensional media have received much less attention [22].

The geometrically nonlinear behavior of heterogeneous media can be described by the Green-Lagrange strain tensor [12]. During the past years, different types of higher-order continuum models have been developed to describe their effective response [23–25].

It needs to be noted that the acoustic properties of metamaterials cannot be separated from the homogenization approach used for the computation of their effective properties [26–28]. In order to adequately describe frequency-dependent homogenized properties, it is necessary to fully generalize the theory of homogenization to the dynamic regime in a manner that allows for an accurate computation of the dispersion relation at any wavenumber and frequency. Analysis of the type is usually referred to as “dynamic” or higher-frequency” homogenization [29–32].

The wave propagation characteristics of non-linear systems with a periodic inner structure have been commonly explored using a perturbation approach detailed in Ref. [33]. The method is effective for the computation of not only weakly non-linear systems [34], but also in the case of strongly non-linear responses, as illustrated for different examples of discrete systems [35]. A review of different perturbation approaches for the analysis of wave propagation in weakly nonlinear phononic systems is provided in Refs. [36–41]. The perturbation method makes use of the Bloch waves of an undamped phononic medium as basic functions to provide approximate closed-form expressions for the complex eigenvalues and eigenvectors of the damped system [42]. What is more, asymptotic approaches to predict the nonlinear dispersion relation for both one- and two dimensional periodic arrangements that capture cubic, non-linear interactions can be found in Refs. [43,44]. The arising cubic (contrary to quadratic) terms appear first in the predicted amplitude-dependent dispersion corrections (or shifts). Asymptotic approaches have commonly used cubic order related terms as external forcing vectors, treated independently from the structures' inner element non-linear stiffness contributions which are neglected [45]. Furthermore, long wavelength continuum approximations, based on a multiple scales perturbation method have been used to analyze nonlinear chains subject to external forces [46]. Moreover, the non-linear dispersion characteristics of hyperelastic media have been elaborated, making use of the second order expansion of the frequency and displacement components within the Linstedt-Poincaré perturbation framework [47].

The characterization of the dynamic behavior of structural systems is of primal importance not only in the realm of architectured media, but also in the structural analysis, design and seismic evaluation of civil engineering constructions [48]. The latter are commonly based on a linear system eigenvalue analysis that typically disregards nonlinear effects such as the wave amplitude-dependent dispersion diagram [49]. What is more, code design specifications commonly rely on the first principal eigenmode [50], with practical design methodologies to exhibit different limitations [51].

In this work, we analyze the effect of the consideration of the full Green-Lagrange strain on the dispersion diagram of architectured materials. To that scope, using the Euler-Lagrange equations of motion, we derive the systems' equilibrium equations in a two-dimensional context. Focusing in the present work on periodic structures made by the repetition of a representative unit cell and in view of the system discretization, we employ two-node Euler Bernoulli beam elements (Section 2.1). We subsequently use the Lindstedt-Poincare perturbation method to derive the system of equations characterizing the structure's wave propagation attributes, accounting for the Green-Lagrange induced, higher order kinematic contributions (Section 2.2). In Section 3, we apply the elaborated methodology to compute the dispersion relation diagram of different, two-dimensional artificial materials, architectured with hexagonal, re-entrant hexagonal (Section 3.1), as well as square and triangular-shaped unit cells (Section 3.2). We subsequently comment on the effect of the higher order inner element kinematics on the dispersion diagram, discussing the role and impact of wave amplitude, wave propagation direction and unit-cell design in Section 4.