Journal of Sound and Vibration 431 (2018) 295–303

Contents lists available at ScienceDirect

Journal of Sound and Vibration

journal homepage: www.elsevier.com/locate/jsvi

Calculation of Lyapunov exponents in impacted beam on distributed contact

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ARTICLE INFO

Article history:
Received 5 March 2018
Received in revised form 24 May 2018
Accepted 10 June 2018
Available online 15 June 2018
Handling Editor: L.N. Virgin

Keywords:
Chaos
Beam
Lyapunov exponent
Vibro-impact
Distributed contact

ABSTRACT

This paper presents the calculation of Lyapunov exponents in a beam subjected to distributed impact. In this case, discontinuity sporadically occurs over the contacted area. The impact area is spatially discretized and the transition condition at impact instance is applied to all impact nodes in the finite element manner. The solution of the continuous vibration in the impacted beam is expressed in the modal expansion form. Then, the spectra of Lyapunov exponents in the periodic and chaotic motion are estimated for the truncated number of modes. Numerical results show that the calculated largest Lyapunov exponent agrees with the bifurcation plot.

1. Introduction

Vibro-impact problem has been widely studied in many mechanical applications such gear system. The related bifurcations such as grazing bifurcations and chaotic phenomena were broadly investigated over the several decades by using a simple spring-mass model [1–9]. The impact motion of the cantilever beam with a tip mass was often modeled as a lumped mass model as well [10–13]. Alternatively, it was expressed in the form of mode discretization in the continuous beam system using Galerkin projection, then, a three-mode approximation was found to describe the motion sufficiently [14,15].

The spectrum of Lyapunov exponents is one fundamental tool for analyzing the nonlinear dynamical system. The Lyapunov exponents of the smooth dynamical system can be successfully calculated by using the Benetinn et al. [16] and Wolf's algorithm [17]. However, it cannot be applied to the non-smooth dynamical system such as the vibro-impact model because the Jacobian is not defined at the non-smooth point. Several methods for calculating the Lyapunov exponents of the non-smooth systems were proposed such as synchronization process [18], transcendental map [19], Poincare mapping method [20] and Muller's linearized method [21]. The synchronization method can be used only when there are only two system variables. The transcendental and Poincare mapping methods use the mapping approach on transition from one solution to the other at discontinuities. Muller's method introduced the transition condition of the linearized system at discontinuity. In principle, the mapping and Muller's methods may be applied to the higher degree-of-freedom system with impact.

In many mechanical applications, the system is not as simple as a point contact model. For example, brake disc is subjected to distributed friction contact over the finite contact area [22–25]. Recently, it draws attention from the calculation of the spectrum of Lyapunov exponents in the non-smooth dynamical system with distributed contact problem [26]. Particularly,

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https://doi.org/10.1016/j.jsv.2018.06.023

the Lyapunov exponents for the more complex system even in the finite element method are expected to be calculated as the computation speed is increased. It requires to calculate the Lyapunov exponents in the presence of the discontinuity at many contact nodes. However, it is extremely difficult to estimate the spectrum of Lyapunov exponents in the higher degree-of-freedom system with distributed impact area.

To the author's knowledge, this is the first work to present the calculation of the spectrum of Lyapunov exponents for the non-smooth system with distributed impact. In this study, an impacted Euler-Bernoulli beam is under the subject of the calculation of Lyapunov exponents. The Muller's method is applied by introducing the relevant transition function for the distributed impact. The Euler-Bernoulli beam on the elastic foundation as the distributed 'soft' impact is used in this study. By the proposed method, it is shown that the complex continuous vibration system with non-smooth distributed impact can be approximately estimated.

2. Beam under distributed impact

The simply-supported Euler-Bernoulli beam is located near the ground-fixed elastic foundation with a clearance r and the left and right end locations (b_1 and b_2) as shown in Fig. 1. The elastic foundation takes a role of soft contact where the impact law follows the piecewise linear stiffness without damping. The beam is excited at the central point with an excitation frequency ω. It provides distributed impact on the elastic foundation when the beam is shaken.

The beam system under distributed impact can be described in the Hamilton's principle as:

$$\int_{t_1}^{t_2} \{\delta T - \delta U + \delta W_{\mathrm{c}} + \delta W_{\mathrm{ex}}\} \mathrm{d}t = 0 \tag{1}$$

where the virtual kinetic and strain energies of the beam are given by:

$$\delta T = \rho \mathrm{A} \int_0^L \dot{u} \cdot \delta u \mathrm{d}x, \tag{2}$$

$$\delta U = EI \int_0^L u_{,xx} \cdot \delta u_{,xx} \mathrm{d}x, \tag{3}$$

and where ρ, A and EI are the density, cross-sectional area and flexural rigidity of the beam.

The virtual work due to the distributed impact and external point load is expressed in the form:

$$\delta W_{\mathrm{c}} = -\int_{b_1}^{b_2} F_{\mathrm{c}} \delta u \mathrm{d}x, \tag{4}$$

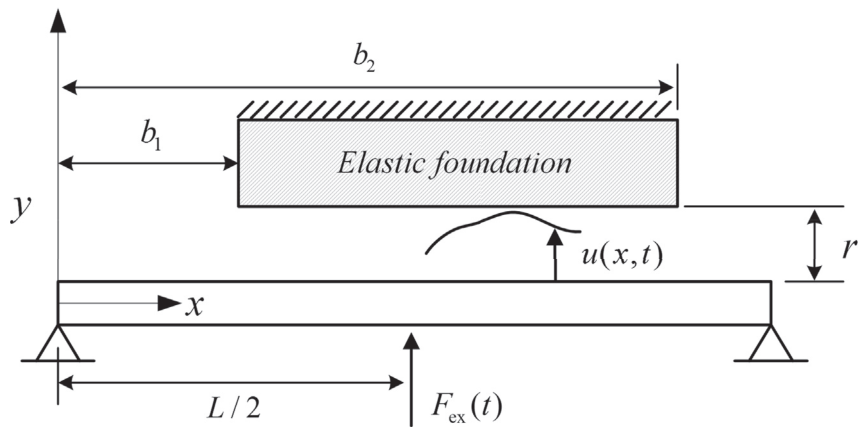

Fig. 1. Configuration of the impacted beam over the elastic foundation.