Journal of Sound and Vibration 429 (2018) 287–304

Contents lists available at ScienceDirect

Journal of Sound and Vibration

journal homepage: www.elsevier.com/locate/jsvi

Vibration-based estimation of beam boundary parameters

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ARTICLE INFO

Article history:
Received 1 September 2017
Revised 24 April 2018
Accepted 11 May 2018
Available online XXX
Handling Editor: I. Trendafilova

Keywords:
Beam theory
Boundary conditions
Tension
Estimation
Rayleigh's quotient
Nonlinear regression

ABSTRACT

Two methods are suggested for using measured vibrations to estimate linear boundary stiffness and damping for beams, while simultaneously estimating axial tension. Estimation is performed by fitting model boundary parameters to measured modal vibration data. The methods are validated using simulated and experimental data, and shown to be accurate when boundary parameters are not extreme, i.e. representing either zero stiffness or compliance.

1. Introduction

We suggest two methods for estimating parameters describing boundary conditions for elastic beams using measured vibrations. Theories used in this paper are derived from beam theory and modal analysis [1,2]. The motivation is the need to estimate tension of beams from vibration data, while taking into account beam boundaries not being ideal, i.e. bolted, riveted, welded, not fully clamped etc. A specific example of the possible use of a simultaneous tension and boundary condition estimation is the problem of estimating bolt tension or tightness from vibration data [3], as bolt tension change the contact stiffness at endpoints.

Methods exist for using vibration data to estimate tension of beams [4–7], but few include estimation of boundary conditions. Methods for estimating axial tension when boundary conditions are unknown exists, for example a method which uses five mode shape measurements of a single mode to estimate four mode shape coefficients and tension [8], but without estimating boundary parameters.

Boundaries have both stiffness and damping. For damping estimation with real-world beams, difficult aspects of friction such as hysteresis may be a problem, even when estimating linearized boundary damping. Hysteresis can be modeled with a linear restoring force using a Hilbert transform [9] (loss factor hysteresis), but accuracy and physical interpretations is lost.

General system identification theory for dynamical systems can be used for estimation of unknown parameters [10–12]. Many identification procedures exists, including the *restoring force surface* method [13,14], which can be used with time signals to estimate both nonlinear and linear term parameters; the field of control theory has its own estimation procedures based on

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https://doi.org/10.1016/j.jsv.2018.05.016

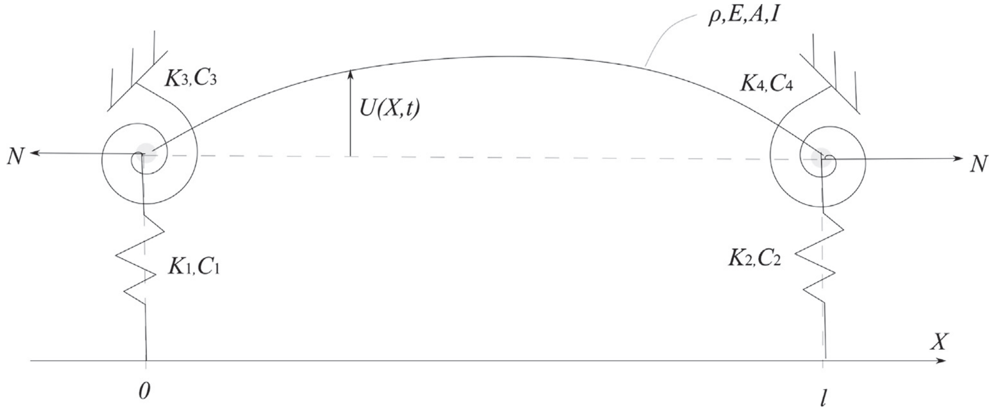

Fig. 1. Beam in tension with springs and dampers at boundaries.

minimal noise principles [15], designed to be real time during measurements.

Linear and nonlinear parameter estimation theory [16] is relevant for estimating boundary parameters. Estimation of beam parameters using least squares minimization with experimental data has been demonstrated for specific cases [17,18], but not for arbitrary boundaries.

A method for simultaneous estimation of axial tension and boundary parameters of beams is needed. In this work we suggest two methods: Method 1 fits measured natural frequencies, damping coefficients and mode shapes to Rayleigh's quotient using linear regression, while Method 2 fits measured natural frequencies and/or mode shapes to the solution of an eigenvalue problem using nonlinear regression. Method 2 estimate boundary stiffness parameters and tension, while Method 1 also estimates boundary damping parameters, but requires mode shapes.

Section 2 presents the beam boundary model used for analysis. Section 3 presents Method 1 and Section 4 presents Method 2. Section 5 validates the two methods using simulated and experimental data. Section 6 concludes on the validity of proposed estimation methods.

2. Mathematical model

Fig. 1 shows a beam in tension, with linear and rotational springs [19] and dampers at its boundaries. The beam has length l, and axially uniform mass density ρ, Young's modulus E, area moment of inertia I and cross-section area A. The beam performs transverse vibrations $U(X,t)$ in the plane of the paper, where $X \in [0;\, l]$ is the axial coordinate, and t the time; it's curvature can be approximated as the second derivative $U''(X,t)$, and the axial tension N is axially uniform and constant. Transverse and longitudinal boundary conditions consists of transverse and rotational springs K_{1-4}, dampers C_{1-4}, and prescribed axial tension N.

2.1. Equation of motion

Nondimensional quantities are introduced:

$$x = \frac{X}{l},\; u(x,\tau) = \frac{U(X,t)}{l},\; \tau = \omega_0 t,\; \omega_0 = \sqrt{\frac{EI}{\rho A l^4}},$$
$$p = \frac{Nl^2}{EI},\; k_1 = \frac{K_1 l^3}{EI},\; k_2 = \frac{K_2 l^3}{EI},\; k_3 = \frac{K_3 l}{EI},\; k_4 = \frac{K_4 l}{EI},$$
$$c_1 = \frac{C_1 l^3 \omega_0}{EI},\; c_2 = \frac{C_2 l^3 \omega_0}{EI},\; c_3 = \frac{C_3 l \omega_0}{EI},\; c_4 = \frac{C_4 l \omega_0}{EI}. \tag{1}$$

Nondimensional position and time are $x = X/l$ and $\tau = \omega_0 t$, where ω_0 is a characteristic angular frequency. The nondimensional deflection is $u(x,\tau)$ and its derivatives will be denoted u' w.r.t. x and $\dot{u}$ w.r.t. τ. The nondimensional tension p describes a ratio between transverse beam stiffness originating from tension and from bending, and includes information about beam slenderness, since $l^2/I = s_r^2/A$, where $s_r = l/\sqrt{I/A}$ is the slenderness ratio. Boundary springs and dampers are nondimensionalized into k_{1-4} and c_{1-4}.

Bernoulli-Euler's equation of motion for deflections $u(x,\tau)$ [1] is, in nondimensional form

$$\ddot{u} + u'''' - pu'' + \mathrm{D}\dot{u} = 0, \tag{2}$$