Journal of Sound and Vibration 428 (2018) 168–178

Contents lists available at ScienceDirect

Journal of Sound and Vibration

journal homepage: www.elsevier.com/locate/jsvi

Structural intensity analysis of flat plates based on digital stroboscopic holography measurements

F. Pires [a, b, *], P.G.G. Muyshondt [a], W. Keustermans [a], S. Vanlanduit [a, b], N.B. Roozen [c], J.J.J. Dirckx [a]

[a] *University of Antwerp, Laboratory of Biophysics and Biomedical Physics, Department of Physics, Groenenborgerlaan 171, 2020 Antwerp, Belgium*
[b] *Vrije Universiteit Brussel, Department of Mechanical Engineering, Pleinlaan 2, 1050 Brussels, Belgium*
[c] *Katholieke Universiteit Leuven, Laboratory of Acoustics, Soft Matter and Biophysics, Department of Physics and Astronomy, Celestijnenlaan 200D, 3001 Leuven, Belgium*

ARTICLE INFO

Article history:
Received 31 July 2017
Received in revised form 1 May 2018
Accepted 5 May 2018

Handling Editor: I. Lopez Arteaga

Keywords:
Digital stroboscopic holography
Source and sink localization
Spatial Fourier Transform
Structural intensity
Energy transmission

ABSTRACT

The analysis of structural intensity vector fields has shown to be a practical approach to characterize the energy flow in plate-like structures and to localize regions of injected or absorbed power. Such an analysis is performed by differentiating measured velocity or displacement fields of a sample. The quality of such a study strongly depends on the spatial resolution of the deformation data and its signal-to-noise ratio. The digital stroboscopic holography concept is presented in this work and used as a tool to record such deformations on a flat plate, which is in direct contact with a shaker and a damper. The current set-up could provide the recording of displacement fields at a high spatial resolution showing little corruption by noise. These conditions permitted the measurement of deformation patterns containing short wavelengths, which were later used as inputs for the structural intensity analysis. By calculating the spatial derivatives of several out-of-plane displacement fields, the energy flowing through the plate was estimated and the position of the external devices (shaker and damper) could be identified by locating regions of the energy dispersion or convergence. The location of these specific regions was made even clearer by calculating the divergence of the energy flow.

1. Introduction

Structural intensity (SI) is a concept that describes the energy being transported by elastic waves in a structure. A commonly studied case is the energy flowing through thin plates. If the necessary simplifications are assumed, it is possible to assess the transmission of energy using the measurements of the sample's out-of-plane motion. In the past, several attempts have been made to reach this goal using contact method approaches, such as accelerometers [1,2]. The main limitation of such an approach is that it alters the system's behavior due to the added mass of the transducer. Later, non-contact methods which do not add mass to the measured structure were used to detect surface motions, i.e., the Laser Doppler Vibrometry (LDV) [3–6], acoustic-holography [7], electronic speckle interferometry [8] or holographic interferometry [9]. These techniques are

---

[*] Corresponding author. Groenenborgerlaan 171, G.U.341, 2020 Antwerpen, Belgium.
E-mail address: felipe.pires@uantwerpen.be (F. Pires).

https://doi.org/10.1016/j.jsv.2018.05.010

not only non-contact but also allow the measured displacement or velocity fields to be recorded with higher spatial resolution than the mentioned pioneering studies. Moreover, the grid sizes from non-contact measurements found in literature vary from 165 (15 × 11) [10] to 15876 (126 × 126) [8] measurement points.

In turn, the spatial derivatives of the out-of-plane displacement fields provide the necessary inputs for the assessment of SI and the localization of sources and sinks. Many studies have already been carried out to estimate the energy flow in the form of vector fields or by calculating its divergence, which allows clear visualization of vector's dispersion or convergence [4,5,11–15].

The Digital Stroboscopic Holography (DSH) is presented in this paper as another alternative among the available non-contact methods to study the structural intensity. A set-up that measures the out-of-plane displacement of a harmonically excited thin flat plate by using this technique is presented. After calibration, a total of 305.373 measurement points (411 × 743) were achieved and the images were processed for further analysis.

2. Theory

2.1. Power and structural intensity

The expressions regarding the definition of power [3,15–18] and structural intensity in plates [12–14,19–21] are well documented in literature and are therefore just briefly reviewed here. The mean power is defined as the time-averaged product of generalized forces and related in-phase velocities. In the particular case where it is desired to investigate the transferred power injected in the structure at an excitation point and by assuming that it behaves harmonically, the mean transmitted power can be directly calculated by

$$P = -\frac{1}{2}\mathrm{Re}\left\{\tilde{F}\cdot\tilde{v}^{*}\right\}, \tag{1}$$

where $\tilde{F}$ is the force with which the structure is excited in the normal direction, $\tilde{v}$ is the out-of-plane velocity at the point of excitation, $\sim$ denotes the complex representation of the mentioned field and $*$ denotes the complex conjugate operator. Depending on the value which is extracted from Eq. (1), it can be interpreted if the power is either injected or absorbed at the excitation point. In the particular case where power is injected into the system, the absolute phase difference between the force $\tilde{F}$ and the velocity is less than $\frac{\pi}{2}$ radians. The smaller the phase's difference between these 2 quantities, the greater the injected power.

On the other hand, if the absolute relative phase is greater than $\frac{\pi}{2}$ radians, power is absorbed from the system. In cases where the phase difference of the force and velocity is nearly π radians, the measured point is considered to have strong viscous damping behavior. Furthermore, if $\tilde{F}$ and $\tilde{v}$ are totally out-of-phase, i.e. with a phase difference equal to $\frac{\pi}{2}$ radians, no power is transmitted, which means that the measured point behaves as a combination of a mass and spring.

In flat plates, the power transmission is caused by the interaction between internal generalized forces and velocities. One way to assess the energy flow within the plate is by assuming that the sample behaves as a Kirchhoff plate. Since the flexural waves are the wave types which most contribute to the vibration energy in plates [9,16], Eq. (1) can be simplified and recast in terms of

$$\mathbf{I} = (I_x, I_y), \tag{2}$$

$$I_x = -(\pi f)\mathrm{Im}\left\{\tilde{Q}_x\cdot\tilde{w}^{*} + \tilde{M}_{xy}\cdot\frac{\partial\tilde{w}^{*}}{\partial y} + \tilde{M}_x\cdot\frac{\partial\tilde{w}^{*}}{\partial x}\right\}, \tag{3}$$

$$I_y = -(\pi f)\mathrm{Im}\left\{\tilde{Q}_y\cdot\tilde{w}^{*} + \tilde{M}_{xy}\cdot\frac{\partial\tilde{w}^{*}}{\partial x} + \tilde{M}_y\cdot\frac{\partial\tilde{w}^{*}}{\partial y}\right\}, \tag{4}$$

where $\tilde{Q}_x$ and $\tilde{Q}_y$ denote out-of-plane shear forces, $\tilde{M}_x$ and $\tilde{M}_y$ the bending moments, $\tilde{M}_{xy}$ the twisting moment, f the excitation frequency, $\tilde{w}$ the out-of-plane displacement field and $\mathbf{I}$ the active SI vector field per unit length (W/m).

The right-hand terms of Eq. (3) and Eq. (4) depend solely on the material properties and on the out-of-plane displacement field $\tilde{w}$ of the plate. If the system is considered to behave in accordance with the Kirchhoff's theory of thin plates, the shear forces and bending moments can be estimated as functions of the out-of-plane displacements:

$$\tilde{Q}_x = -\tilde{D}\left[\frac{\partial^3\tilde{w}}{\partial x^3} + \frac{\partial^3\tilde{w}}{\partial x\partial y^2}\right], \tag{5}$$