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# Transverse impact of a Hertzian body with an infinitely long Euler-Bernoulli beam

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ABSTRACT

We study in detail, using analytical approximations and numerical solution, the transverse impact interaction of a compact body with an infinitely long Euler-Bernoulli beam. The beam is initially stationary, linear and dissipation-free; a Hertzian spring mediates body-beam contact; and the body is otherwise rigid. Impact interaction obeys two nonlinear differential equations with a fractional order derivative. Prior progress on the infinite-beam problem has been limited. Here we completely characterize the possible contact behaviors in terms of one nondimensional number, $S$, which governs separations and sustained contact regimes. For small $S$, there is just one contact phase followed by separation. For large $S$ no separation occurs, and sustained contact occurs with decaying oscillations. For intermediate $S$, separation occurs one or more times, followed eventually by sustained contact. The number of separations can be large over a small range of $S$. A semi-analytical approximation matches well the smaller-$S$ behavior until first separation. A separate asymptotic approximation matches the long-time sustained contact behavior for higher $S$, independent of the intervening number of separations. The two approximations work on overlapping ranges of $S$. Neither approximation captures the multiple separations of intermediate $S$, where we use full numerics with a published recipe for fractional order systems. The numerics match the abovementioned analytical results.



## 1. Introduction

Impacts involving flexible bodies, like plates and beams, have been studied by many authors over many years; see the classic text by Goldsmith [1], and recent surveys [2,3]. There is an extensive literature on vibration-dominated contact, wherein aspects studied have ranged from vibrations and waves [4–8], coefficient of restitution [9,10], energy dissipation or transmission [11], local deformation [12], and contact detection [13].

Impact between a compact body and an infinitely large slender body constitutes a special limiting case, provides insights into observed behavior even for impacts with large but finite-sized objects, and in any case presents an academic problem that is interesting in its own right. This paper presents a study of such an impact.

A particular problem of historical interest within this category is the impact of a Hertzian sphere with a large thin *plate*, which is analytically simpler than impact with a beam. Raman [14] carried out experiments for a sphere striking an extended thin plate, while Zener [15] did the analysis assuming an infinite plate and obtained a good match with Raman's experiments. Zener showed that the impulse response of an infinite plate is $y = P\alpha$, where $y$ is displacement of the point where an instantaneous impulse $P$ acts, and $\alpha$ is a constant dependent on plate parameters. This impulse response, $y = P\alpha$, is the same as that of a single dashpot.

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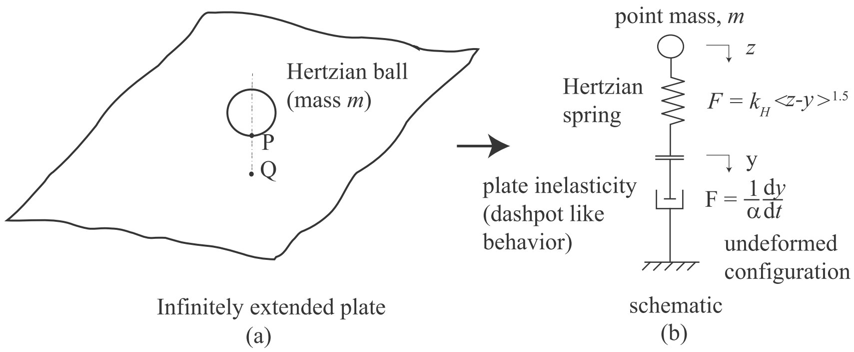


**Fig. 1.** (a) A Hertzian sphere striking a thin infinite plate, and (b) its dynamic equivalent. The impulse response of the plate is a constant, same as that for a dashpot. $y$ and $z$ denote the displacements of the plate's contact point and the sphere's center respectively.

Thus, the lateral impact of a Hertzian sphere with a large plate is mathematically the same as that of a point mass falling on a dashpot with a mediating Hertzian spring in between (see Fig. 1).

In this paper we consider impact between a compact Hertzian object and an infinite beam, which is more complicated than the contact between a ball and plate, as mentioned above. Schwieger [17], following [1,16], noted that elastic central impact behavior on a sufficiently long beam is independent of both the beam's boundary conditions as well as its length. Along the lines of Zener [15], Schwieger [17,18] then obtained the relation for the short-time deflection of a finitely long Euler-Bernoulli beam under impact loading (an approximate solution); an extra factor of 2 was corrected in the later paper. The same impulse response function for an infinite beam (an exact solution) was obtained in Refs. [19,20] using the Fourier transform, and in Refs. [21,22] directly in the time domain. The key point is that a beam's impulse response is proportional to $\sqrt{t}$. Schwieger [17] also presented some experiments with single contact phases.

More recently, Yigit & Christoforou [23] presented a general numerical study of the impact of a mass on a composite beam and plate with a *linearized* contact stiffness. These authors proposed two nondimensional numbers using which the overall contact behavior might be characterized into different regimes, identifying single contact at two extremes, with passing remarks on the intermediate regime where multiple impact events are possible.

In this paper, we present significant progress beyond [17,18,23]. In particular, we present a detailed study of the lateral impact of a compact Hertzian body with an infinitely long Euler-Bernoulli beam, characterizing the full range of possible contact dynamic behaviors, from impacts with a single contact phase followed by separation, through impacts with more than one contact phase followed by sustained contact, to impacts with no separation at all.

We now turn to equations of motion.

### 1.1. Equations of motion

The equations we use are similar to those in Refs. [17,18].

See Fig. 2. A solid body of mass $m$ strikes a horizontal initially stationary beam. Hertz contact is assumed. The beam has uniform mass per unit length $\overline{m}$ and flexural rigidity $EI$. The displacement (upwards) of the beam's contact point is taken as $y(t)$. The distance (upwards) of the notional point of contact of the body from the beam's notional undeformed surface is taken as $z(t)$

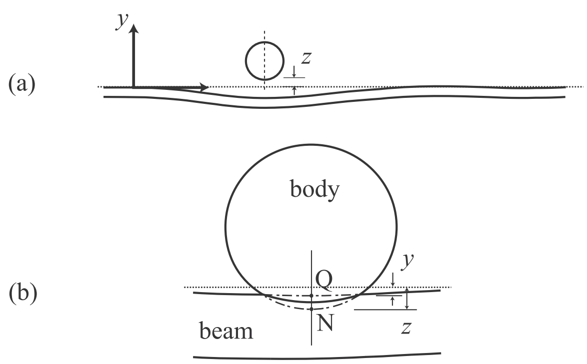


**Fig. 2.** (a) Transverse impact of a compact body on an infinite Euler-Bernoulli beam. The displacement of the beam and the body are referred to as $y(t)$ and $z(t)$ respectively. (b) The body-beam contact. The solid lines show the actual configuration at contact. N is the notional contact point on the undeformed body. Q is the notional contact point on the beam. The distance from N to Q is the compression (positive in the sense shown); and a contact force exists when the compression is positive.