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Cross-frequency and band-averaged response variance prediction in the hybrid deterministic-statistical energy analysis method

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ABSTRACT

The hybrid deterministic-statistical energy analysis method has proven to be a versatile framework for modeling built-up vibro-acoustic systems. The stiff system components are modeled deterministically, e.g., using the finite element method, while the wave fields in the flexible components are modeled as diffuse. In the present paper, the hybrid method is extended such that not only the ensemble mean and variance of the harmonic system response can be computed, but also of the band-averaged system response. This variance represents the uncertainty that is due to the assumption of a diffuse field in the flexible components of the hybrid system. The developments start with a cross-frequency generalization of the reciprocity relationship between the total energy in a diffuse field and the cross spectrum of the blocked reverberant loading at the boundaries of that field. By making extensive use of this generalization in a first-order perturbation analysis, explicit expressions are derived for the cross-frequency and band-averaged variance of the vibrational energies in the diffuse components and for the cross-frequency and band-averaged variance of the cross spectrum of the vibro-acoustic field response of the deterministic components. These expressions are extensively validated against detailed Monte Carlo analyses of coupled plate systems in which diffuse fields are simulated by randomly distributing small point masses across the flexible components, and good agreement is found.

1. Introduction

Effective vibro-acoustic analysis tools are essential for the design of engineering systems with good noise and vibration performance, both in terms of safety and human comfort. Ideally such tools would be applicable over the entire frequency range of interest, but in practice, the workability of specific methods is restricted to a limited frequency region. Straightforward application of deterministic field-based analysis techniques such as finite element or boundary element analysis is possible at low frequencies, as then the wavelength of deformation is long in all system components and the vibro-acoustic response is dominated by a limited number of eigenmodes [1,2]. However, as the frequency increases, the wavelength of deformation becomes short in the flexible components of the built-up system, resulting in refined meshes and a large computation cost [3]. At the same time, the vibro-acoustic field response at a specific location in such a flexible component becomes highly sensitive

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to the possible presence of small entities with a wave scattering effect, such as local inhomogeneities or small objects attached to the component [4,5]. As a result, a deterministic analysis at high frequencies is not only painstaking due to the high level of detail and the related vast amount of information that are required, but also less relevant, since the large effect of small changes in the system configuration compromises the robustness of such analysis.

In contrast to what might be expected, one can exploit both the sensitivity of the local response of a flexible system component to small wave scattering entities and the fact that precise information on these entities can often not be obtained at reasonable cost, for setting up an effective high-frequency component model. This is achieved by modeling the wave field due to random wave scattering directly, rather than modeling the physical mechanisms behind the wave scattering. If the randomness of the wave scattering is sufficiently large such that a state of maximum uncertainty, termed maximum entropy, is reached, then the related wave field is called diffuse [6,7].

The notion of a diffuse field dates back to Sabine's reverberation theory [8], but it has evolved over time with an increasing level of sophistication. The basic definition is the following [9–12]: a diffuse wave field is a random field, composed of a large number of statistically independent plane waves, the spatial phase of which is uniformly distributed and independent from the amplitude. In this basic definition, a diffuse field is a Gaussian random field, which is entirely defined by its mean and spatial covariance functions. When the field is harmonic (i.e., the result of an excitation at a single frequency), the spatial covariance function follows directly from the definition [13]. In particular, the variance of the wave field is the same at all points, so the mean energy density is uniformly distributed across the system.

Such a model can only describe the behavior of a random system in an exact way when it has diffusely reflecting boundaries, i.e., boundaries that reflect an incident plane wave in a random direction that is statistically independent of the direction of incidence. When boundary reflections are specular rather than diffuse, the mean energy density will not be uniform anymore because of interference between incoming and reflected sound waves, but this effect can be accounted for [14]. Further generalization is possible by establishing a harmonic linear relationship between the mean total energy in the diffuse field and the mean squared amplitude of the blocked interface reverberant forces, i.e., the forces that are caused by the diffuse field at the blocked deterministic interface degrees of freedom. This relationship is termed the diffuse field reciprocity relationship [7]. Its evaluation requires the computation of the radiation impedance matrix of the deterministic interface degrees of freedom at the considered frequency.

The notion of diffuseness discussed so far makes use of a wave description of a vibro-acoustic field. It was unclear how the required wave field properties translate to conditions on the modal properties until Lyon [6] made a connection with the theory of maximum entropy (in the probabilistic sense of maximum uncertainty): when a system has diffusely reflecting boundaries, it caries a minimum amount of information about these boundaries since its statistical properties are invariant to translation and rotation. In a modal description, a vibration or acoustic field is maximally uncertain (in other words, diffuse) if all excited modes are equally energetic in an ensemble averaged sense and if the mode shapes are statistically independent from each other [6]. The diffuse field reciprocity relationship can also be derived based on this modal definition [15].

A further generalization of the diffuse field concept has been driven by the search for an accurate quantification of the variance of the energetic response. For a modal description of a diffuse field in particular [6], the second-order statistics of the natural frequency spacings are needed and these are not prescribed by the conventional diffuse field model. Weaver [16] found that, in high-frequency regime, these statistics conform to those of the eigenvalues of a Gaussian Orthogonal Ensemble (GOE) random matrix [17], rather than to the other models that had been proposed earlier [18,19]. The GOE model is a substantial generalization of the conventional diffuse field model. The formal connection is that, in the GOE model, the mode shapes are independent Gaussian random fields. Assumptions other than on the nature of the statistics of the natural frequencies and mode shapes are not needed. Recent GOE-based results include derivations of the variance of frequency response functions and energy densities of a homogenous reverberant system [20–22] and the variance [23,24] and probability distribution [25] of its total energy. The GOE model also enabled a generalization of the diffuse field reciprocity relationship by dropping the assumption of equipartition of energy between the modes [15].

The analysis of the total vibrational energy of and the power flows between diffuse components of a built-up system is termed statistical energy analysis (SEA) [26]. The key to modern SEA is the diffuse field reciprocity relationship because it enables the computation of coupling loss factors in a rigorous and straightforward way, and therefore it resolves the problems related to ad hoc approaches that make a distinction between resonant and non-resonant transmission [27]. It also allows combining reverberant system component models with other component models, such as finite element or boundary element models, within a single, hybrid framework of analysis. The resulting hybrid deterministic-SEA method [28] can therefore be also employed for the analysis of built-up systems in the medium frequency range, where the stiff components of the built-up system (e.g., the beam-column frames of a building) still exhibit long-wavelength deformation while the flexible components (e.g., the floors and walls of a building) already exhibit short-wavelength deformation.

The hybrid deterministic-SEA method has been extended such that not only mean energetic response quantities can be computed, but also their variance [29]. This variance represents the uncertainty that is due to the assumption of a diffuse field in the flexible components of the hybrid system. At present, only the variance of the harmonic energetic response can be computed, i.e., the variance of the system's response at a single frequency. However in technical acoustics, energetic responses are usually averaged or integrated over frequency bands because such integration correlates well with human perception of sound and vibration, and because it results in a reduced uncertainty of the energetic quantities.

Therefore, in the present paper, the hybrid method is extended such that also the ensemble variance of the band-averaged system response can be computed. After the introduction of the necessary definitions and existing results (section 2), the devel-