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Vibration around non-trivial equilibrium of a supercritical Timoshenko pipe conveying fluid

Xia Tan [a], Xiao-Ye Mao [a], Hu Ding [a, b, *], Li-Qun Chen [a, b, c]

[a] Shanghai Institute of Applied Mathematics and Mechanics, Shanghai University, Shanghai 200072, China
[b] Shanghai Key Laboratory of Mechanics in Energy Engineering, Shanghai University, Shanghai 200072, China
[c] Department of Mechanics, Shanghai University, Shanghai 200444, China

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Vibration characteristics of pipes conveying fluid in the super-critical range are investigated by using Timoshenko beam theory for the first time. Generalized Hamiltonian principle is applied to derive the nonlinear transverse vibration governing equation. The non-trivial static equilibrium configuration and critical flow velocity of the pipe are analytically deduced. These analytical results are verified by using the finite difference method. Compared with Euler-Bernoulli flow pipeline, it is found that the equilibrium configuration of Timoshenko pipe is larger. In the supercritical regime, natural frequencies of the Timoshenko flow pipe are produced by the Galerkin truncation method. Numerical examples illustrate that vibration characteristics of the pipe are highly sensitive to length, thickness, shear modulus and velocity. The relative difference between the two pipe models is influenced by the velocity of the flow and is likely to exceed 100%. In general, this work found that the flow velocity makes the Timoshenko beam theory even more needed for researching vibration properties of pipes conveying fluid, especially at a high velocity.

1. Introduction

Vibration of the conveying pipe caused by the flow is very common in actual engineering. Pipes will vibrate violently and even cause damage when velocity is too high. With more and more pipelines are involved in projects such as oil, water and gas transmission, many scholars began to paid their attention to vibration characteristics of pipes conveying fluid. Gregory experimentally investigated the global behavior and stability conditions of unstable oscillation for a cantilever pipe conveying fluid [1]. Païdoussis first studied the dynamic stability of flexible pipes under a constant or small harmonic flow velocity [2]. Plaut determined that the axial loading would affect the stability of a fluid-conveying pipe [3]. The papers above all researched the linear eigenvalue formulas of the dynamic stability of pipes conveying fluid. Païdoussis summarized linear dynamic problems of the transmission pipelines [4]. In order to satisfy the requirements of the engineering field, the research on dynamics of pipes conveying fluid had turned to non-linearity.

Considerable research efforts have been devoted to the nonlinear dynamics of pipes conveying fluid. The precondition for studying these dynamic behaviors was to obtain governing equations. Many scholars deduced the nonlinear equations for pipes conveying fluid successively [5–7]. Thurman and Mote [8] and Holmes [9] investigated nonlinear vibration of

* Corresponding author. Shanghai Institute of Applied Mathematics and Mechanics, Shanghai University, Shanghai 200072, China.
E-mail address: dinghu3@shu.edu.cn (H. Ding).

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pipes containing flowing fluid caused by axial elongation. Thompson considered the large curvatures of pipes [10]. Nonlinear vibration of pipes conveying fluid leads to many interesting dynamic phenomena. Rousslete [11] studied flutter of articulated pipes. Holmes [9] and Bajaj [12,13] investigated the bifurcations induced by flow. McDonald considered the local and global bifurcation of a pulsating fluid pipe under the condition of 0: 1 internal resonance [14,15]. Xu studied the internal resonance and modal exchange of a horizontal cantilever flow tube [16,17]. Lin discussed chaotic and quasi-periodic motions of a curved pipe [18]. Ibrahim [19,20] reviewed the investigation of pipes conveying fluid in the past 60 years. Ghayesh et al. researched the three-dimensional dynamic characteristics of a cantilevered pipe [21,22]. Chang et al. studied the multi-dimensional oscillations of cantilever flow tubes with and without end masses [23]. Based on the Euler-Bernoulli beam theory and Hamilton's principle, Huo et al. established governing equation of a vertically deploying or retracting cantilevered pipe [24]. Ni et al. studied the free vibration and stability of a pipe-flow model with axially moving supports at both ends using numerical methods [25]. Much work so far has focused mainly on the nonlinear dynamics of sub-critical pipe-flow systems.

When the flow velocity exceeds critical value into supercritical range, the pipeline will be re-stabilized from the zero balance position to a new curved configuration. The vibration properties of pipes conveying fluid in super-critical that differ from that of in sub-critical has aroused the interest of the majority scholars. In the supercritical regime, Zhang and Chen pointed out that the 2:1 internal resonance existed in the fluid-structure interaction system and probed the double-jump phenomenon [26–28]. Mao et al. discussed the forced vibration of a transmission pipe with 3:1 internal resonance by using a multi-scale method [29]. Zhou studied the super-harmonic response and the 1:2 internal resonance of supercritical conveying pulsating fluid pipes [30]. Nonlinear dynamic behaviors of pipe-flow systems in the supercritical range are very different from that of in sub-critical range. However, these researches are limited to the Euler-Bernoulli theory.

Several researchers have employed Timoshenko beam theory to research the vibration characteristics of shorter pipes conveying fluid. Huang first established the vibration pipe-flow model based on the principle of Timoshenko [31]. Laithiers and Païdoussis obtained governing equations for the compressed Timoshenko pipe by using the Hamilton's principle [32]. Pramila et al. studied and compared the critical flow speed of Timoshenko pipe models [33]. Lin and Tsai [34] analyzed the vibration behavior of nonlinear Timoshenko pipes by the finite element method. Zhai et al. investigated the dynamic response of the Timoshenko pipe which subject to random excitation [35]. Yu utilized the Timoshenko beam theory to study the flexural vibration band gap [36]. Li and Hu introduced a method for predicting the critical flow velocity of fluid-conveying magneto-electro-elastic pipe [37]. Notably, there are few papers on applying Timoshenko beam theory to study the dynamics of supercritical flow-pipe models.

In the present work, the Timoshenko beam theory is applied to analyze the dynamic properties of supercritical pipe conveying fluid. Based on the generalized Hamilton's principle, governing equations of nonlinear vibration of the Timoshenko pipe conveying fluid are deduced. The equilibrium configurations are obtained. Governing equations of transverse vibration around the buckled configuration are received by coordinate substitution. Natural frequencies of the supercritical flow-pipe model are calculated. Compared with the Euler-Bernoulli model, the influences of length, thickness, flow velocity and shear modulus on the vibration characteristics are revealed.

2. Mathematical models

A uniform and elastic pipe conveying fluid has be described simply in Fig. 1, where x and y represent axial coordinate and radial coordinate, respectively. $v(x,t)$ denotes the lateral displacement of the pipe. L stands for the length of the pipe and t is time coordinate. At both ends, the pipe is constrained by simple supports. In the pipe, the non-viscous and incompressible fluid flows at a constant velocity of Γ. Physical parameters of the pipe which shown in Table 1 are assumed to be constants, where D and h_0 denote the outer diameter and thickness of the pipe, separately. I is moment of inertia. E is young's modulus. P_0 is the initial tension. ρ_p and ρ_f are pipe density and fluid density, respectively. In order to make the theoretical research more practical, the data selected in Table 1 refer to the open literature [29], which are the parameter values of a polypropylene random (PPR) pipe conveying fluid.

In order to study the nonlinear dynamics of pipe subjected to a high flow velocity more precisely, section rotation and shear deformation of the pipe conveying fluid are considered. Based on Timoshenko beam theory, the generalized Hamilton's principle is applied to derive nonlinear governing equation of the pipe conveying fluid.

The kinetic energy of the pipe is

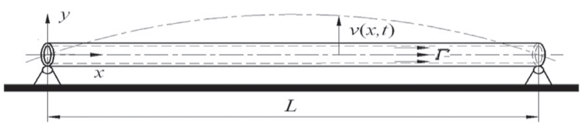

Fig. 1. Model of the pipe conveying fluid.