Journal of Sound and Vibration 427 (2018) 41–62

Contents lists available at ScienceDirect

Journal of Sound and Vibration

journal homepage: www.elsevier.com/locate/jsvi

Nonlinear vibration of transversely-loaded spinning membranes

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ARTICLE INFO

Article history:
Received 21 October 2017
Revised 22 March 2018
Accepted 10 April 2018
Available online 2 May 2018
Handling Editor: W. Lacarbonara

Keywords:
Nonlinear vibration
von Kármán plate theory
Centrifugal force
Heavy spinning disk

ABSTRACT

The paper studies the transverse nonlinear vibration of an isotropic and homogeneous annular membrane spinning at constant angular velocity, under the action of a uniform transverse load. The equilibrium configuration of the membrane, clamped along the inner edge and free along the outer edge, and the natural frequencies of vibration of the membrane are calculated. A Galerkin procedure is used to determine a reduced order model describing the weakly nonlinear vibration of the membrane, and it is shown that near-resonance vibration can be modeled as a single degree of freedom Helmholtz-Duffing oscillator. A detailed study at vibration frequencies close to the fundamental, axisymmetric vibration mode shows a transition from softening to hardening behavior, and jump phenomena or hysteretic behavior depending on the angular velocity, the transverse load, the amplitude of dynamic excitation and the damping ratio. The results are in agreement with dynamic implicit finite element simulations in Abaqus/Standard and direct experimental measurements.

1. Introduction

Membrane structures are widely used for space applications, as they are light and can be packaged tightly for launch. An interesting deployment and stabilization technique for thin membranes is through centrifugal action, as demonstrated by the IKAROS mission [1]. Many applications require nearly flat membranes, in which case it is important to limit the amplitude of vibration of the membrane. This paper presents a study of the influence of an axisymmetric deflection, induced for example by solar radiation pressure, on the nonlinear vibration behavior of a spinning membrane. At sufficiently high angular velocity, the fundamental mode of vibration is axisymmetric, and hence the study is specifically focused on this particular type of mode, which can be excited by shaking the membrane at the center, in the out-of-plane direction.

Extensive research has been performed on the analysis of vibration of thin spinning disks. The linear modes were studied in [2–4]. Ref. [5] was the first to derive the nonlinear dynamic equations governing the deflection of spinning disks using von Kármán plate equations. In the 1990's, studies of spinning disks were motivated by the development of computer memory disks. Refs. [6,7] studied the stability of spinning disks near a critical speed. Ref. [8] focused on the stability of flexible disks loaded by a stationary concentrated transverse load. Studies of nonlinear vibration of thin membranes excited by a concentrated stationary load were carried out in Refs. [9–12]. These studies have established that flat membranes behave like a Duffing oscillator.

In the context of spinning heavy disks, the deflected equilibrium shape and buckling limits of spinning membranes were studied by Refs. [13–15]. Experimental studies were carried out by Refs. [12,16], and linear vibration measurements were performed by Ref. [17].

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https://doi.org/10.1016/j.jsv.2018.04.015

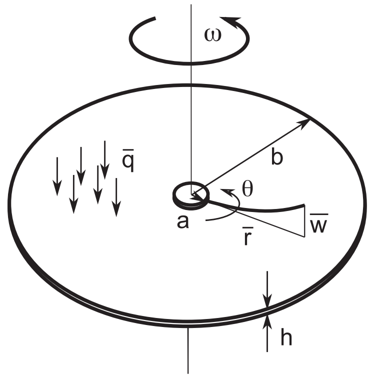

Fig. 1. Notation for spinning annular membrane.

The present authors are not aware of any published work on the nonlinear vibration of membranes loaded by a transverse uniform load and excited near an axisymmetric resonance.

This paper presents a fundamental study of the influence of a gravity-induced deflection on the vibration of a spinning membrane. A circular isotropic clamped-free membrane stabilized by a spinning motion is considered, and its response to near-resonance external excitation is analyzed using a reduced order model, numerical simulations, and experiments. The proposed model captures the effects of the initial deflection of the membrane, and predicts a softening nonlinear behavior at lower angular velocities, which transitions to a hardening behavior at higher angular velocities. These predictions are confirmed both numerically and experimentally.

The paper is arranged as follows. The next section derives the full theoretical model and the reduced order model; results are presented for a wide range of parameters. Section 3 simulates the large-amplitude vibration of a membrane using nonlinear dynamic simulations with commercial finite element software, and compares the simulation results to the reduced-order model. Section 4 presents an experimental setup used to measure the equilibrium shape of spinning Kapton membranes. The first mode shape and the nonlinear vibration of the membrane were measured with two different techniques, digital image correlation and laser scanning vibrometry. A set of experimental results are presented. Section 5 compares the reduced-order model to the experimental results. Section 6 concludes the paper.

2. Theory

The fundamental question to be addressed is, how will the axisymmetric deflection of a spinning membrane, due to a transverse uniform load, affect the nonlinear axisymmetric oscillations of the structure? The membrane is shown in Fig. 1.

A reduced order model for load-deflected membranes is derived in three steps. First, the equilibrium equations are formulated, in Section 2.1.1, then a nonlinear dynamic perturbation around the axisymmetric deflection is imposed, in Section 2.1.2, and finally the reduced order model is derived in Section 2.1.3. A wide range of results for free vibration and forced vibration are then presented in Section 2.2.1 and Section 2.2.2, respectively.

2.1. Analytical formulation

The von Kármán plate theory equilibrium equations for an axisymmetric circular spinning membrane, assuming an isotropic elastic material, have been formulated by Ref. [5]. The derivation starts with the classical plate equation with prestress introduced by in-plane forces, in polar coordinates. Then, in-plane equilibrium is expressed in terms of a stress function and a potential is derived from the centrifugal force. Finally, the compatibility equation is expressed in terms of the stress function using the constitutive equation of Hookean isotropic material as an intermediate step. The equations are expressed as follows in terms of the deflection $\overline{w}$ and stress function $\overline{\Phi}$ [5]:

$$\rho h \frac{\partial^2 \overline{w}}{\partial \overline{t}^2} + D\nabla^4 \overline{w} = L(\overline{w}, \overline{\Phi}) - \overline{c}\frac{\partial \overline{w}}{\partial \overline{t}} - \omega^2 \rho h \left(\frac{1}{2}\overline{r}^2 \nabla^2 \overline{w} + \overline{r}\frac{\partial \overline{w}}{\partial \overline{r}} \right) + \overline{q}(\overline{r}, \theta, \overline{t})$$

$$\nabla^4 \overline{\Phi} = -\frac{1}{2} EhL(\overline{w}, \overline{w}) + 2\rho h(1-\nu)\omega^2 \qquad (1)$$

where $\overline{q}(\overline{r}, \theta, \overline{t})$ is the transverse load, $\overline{c}$ the damping coefficient, ρ the density, E the Young's modulus, ν the Poisson's ratio, $D = Eh^3/12(1-\nu^2)$ is the flexural stiffness of the plate, and the remaining quantities are defined Fig. 1.