Journal of Sound and Vibration 427 (2018) 15–27

Contents lists available at ScienceDirect

Journal of Sound and Vibration

journal homepage: www.elsevier.com/locate/jsvi

Application of the two-loop procedure in multibody dynamics with contact and constraint

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ARTICLE INFO

Article history:
Received 22 January 2018
Received in revised form 9 April 2018
Accepted 12 April 2018

Handling Editor: Erasmo Carrera

Keywords:
Differential algebraic equations
Implicit integration algorithm
Impact
Iteration

ABSTRACT

This paper focuses on the performance of the two two-loop implicit sparse matrix numerical integration (TLISMNI) methods as the numerical solution of index-3 differential algebraic equations (DAEs) of motion arising in stiff multibody dynamics with contact and constraint. The original TLISMNI method proposed for the first time by Shabana and Hussein uses the Newmark method or Hilber/Hughes/Taylor (HHT) method as the implicit integration algorithm [TLISMNI (Newmark/HHT) method]. These two integration algorithms consider the accelerations and Lagrange multipliers as basic unknowns and their numerical accuracy is no more than order 2. In order to have a higher integral precision, we propose a new two-loop implicit procedure based on an extended backward differentiation formula scheme [TLISMNI (EBDF) method] which considers coordinates and velocities as basic unknowns. The whole structure of this proposed method is different from the traditional one due to the different unknowns. Furthermore, we apply these two kinds of methods, TLISMNI (Newmark/HHT) method and TLISMNI (BDF/EBDF) method, to the contact and constraint problems of flexible multibody dynamics while few studies of TLISMNI methods have been done on these non-smooth problems. The Baumgarte method and HHT-α method are also used to solve the same contact and constraint problem, and the advantages and disadvantages of all these methods above are compared in this study. Results show that the proposed method only needs to perform less iterations to satisfy the same tolerance of error than the TLISMNI (HHT) method does. In some cases the Baumgarte method and HHT-α method cannot give reasonable results while the TLISMNI methods perform well when the contact and constraint problems are involved. Moreover, the numerical experiment also indicates that the numerical damping property of the HHT method employed in both the TLISMNI method and the GGL formulation (used in the HHT-α method) can behave in a different way to damp out high frequency oscillations induced by impact.

1. Introduction

In Applied Mathematics, the differential algebraic equations (DAEs) are a general form of differential equations and are used to describe different physical problems, such as parallel manipulators [1], chemical reaction dynamics and electronic systems. It's always been known that the DAEs present particular characteristics which make their numerical treatment

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https://doi.org/10.1016/j.jsv.2018.04.020
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different, and distinguish them from systems of ordinary differential equations (ODEs) [2]. Normally the dynamic equations of constrained mechanical motion are index-3 DAEs in flexible multibody systems (FMS). The solution of a higher-index DAEs can meet more potential difficulties [3], which can limit the availability of numerical solutions of the index-3 systems. Therefore, it is quite necessary to make a further study on the numerical methods. A usual way to lower the index by transforming the DAEs into ODEs is to differentiate the position constraints with respect to time twice. The transformation of the DAE into an ODE involves replacing the constraints with its time derivative plus initial conditions. However, this usually causes the well-known drift difficulties [3–8]. Several solution techniques have been proposed to solve these difficulties. The methods for solving index-3 DAEs are reviewed in Refs. [3,4]. The comparative studies on various solution techniques for stability and constraint violation suppression are made in Refs. [5,6]. The solution techniques to solve the DAEs mainly include the direct discrete methods, stabilization techniques (such as the famous Baumgarte stabilization method), constraint violation elimination techniques and state-space-based approaches. In most cases, the direct discrete methods lead to a maximal number of coordinates and often result in severely ill conditioned problems. The stabilization techniques attempt to minimize or eliminate the constraint violation mainly by the control theory or a penalty term. How to make the former two methods satisfy constraints at all levels (position, velocity and acceleration) is a key point for the solutions to dynamic problems in contact mechanics of FMS. In contrast to stabilization techniques, the elimination techniques enforce exact satisfaction of the constraints [4]. The state-space-based formulation can reduce the dependent coordinates by identifying and expressing them in terms of the independent coordinates through the coordinate partitioning technique. With this, the system of DAEs can be reduced to a set of state-space ODEs which can be solved by using an implicit integration method [7]. The implicit numerical integration requires solving the nonlinear algebraic equations obtained by the discretization of the ODEs. Then the independent generalized coordinates are obtained as the solution of these algebraic equations. Once all the independent variables are known, the dependent variables can be obtained by solving the constraint equations. The identification of independent coordinates and the solution of ODEs and algebraic equations involved in the solving process have the potential of using well established theory and can be achieved by using reliable numerical techniques. Therefore, the state-space-based method has the advantages of good generality and high stylization, and is especially suitable for the numerical solution of DAEs for multibody system dynamics in the virtual prototyping technology. A state-space-based implicit integration algorithm based on generalized coordinate partitioning is presented in Ref. [8]. The two-loop implicit sparse matrix numerical integration method [TLISMNI (Newmark) method], as a state-space-based method, is proposed for the first time in Ref. [9], and the implementation of the TLISMNI (HHT) method is discussed in Ref. [10]. This method takes the generalized accelerations as unknowns, and it is used widely to solve the second-order initial value problems by using the Newmark method or Hilber/Hughes/Taylor (HHT) method in the multibody system dynamics. A modified TLISMNI algorithm that allows for the implementation of several low order integration formulas is presented, which uses the simple iteration and the Jacobian-free Newton-Krylov methods to improve its efficiency and robustness in Ref. [11]. The numerical accuracy of this algorithm is no more than order 2. Sometimes a high-order integration method is needed for the dynamic problems of FMS. Considering that many numerical solution methods are designed for the first-order ODEs and use the generalized positions and velocities as unknowns, how to make more integration methods adopted in the TLISMNI method and what the optimal structure of TLISMNI method based on these integration methods is should be further studied. A new TLISMNI method based on the implicit Runge-Kutta method is proposed to improve the accuracy and efficiency in Ref. [12]. And the variable-step size TLISMNI/Adams method which can use the 3rd and 4th order implicit Adams methods is proposed in Ref. [13]. This method is proved to be efficient for stiff systems.

There is little information available in literature about the TLISMNI method used in contact constraint problems of FMS, which is a typical case of discontinuities. Although the duration of the impact process is very short, the great influence of this process on the dynamic features of FMS cannot be ignored because of the large intensity and strong nonlinearity. As mentioned in Refs. [14–16], it is well known that impact and friction accompany almost all real behavior, leading to non-smooth dynamics. Many research groups investigate the numerical computation of the Lyapunov exponents for the impact system. Many others focus on the solution techniques for the contact constraint problems of multibody system dynamics. A method is proposed to solve the contact constraint problems and points out that it is necessary to formulate the constraint equations on a position, velocity, acceleration, and velocity increment level in Ref. [17]. In order to avoid the choice of parameters provided in the continuous contact force model (CCFM) [18,19], a partition method based on the contact constraint method (CCM) is provided for the impact dynamics of FMS in Ref. [20]. The CCM can describe the impact process accurately and improve the reliability of simulation results. When the Lagrange's equations of the second kind are used without impact, the dynamic equations of FMS are a collection of ODEs. Due to the added nonlinear constraint equations into the impact process, the dynamic equations of the system are usually index-3 DAEs. Therefore, the ODEs are transformed into a set of DAEs in impact. It makes the dimension of dynamic equations change with the contact state. Discontinuities in the velocities by an abrupt change give rise to numerical difficulties at the initial contact time. Furthermore, the addition of high frequencies stimulated by the contact forces can also bring about great challenges. From a mathematical standpoint, the addition of multi-scale characters and discontinuities will make the original problem much more complex. Therefore, we need to analyze the features of solution techniques used in the contact constraint problems of multibody system dynamics.

In this paper, we use the backward differentiation formula (BDF) scheme and an extended BDF (EBDF), respectively, to broad the selection of integration algorithms used in the TLISMNI method. The TLISMNI (HHT) method and TLISMNI (BDF/EBDF) method are used to solve the impact DAEs obtained using the CCM in order to show their advantages and drawbacks in the contact constraint problems of FMS. The CCFM is also used to describe the same impact dynamic process. Unlike the