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Normal impact of a low-velocity projectile against a taut string-like membrane

Yifei Zhao*, Zhili Sun

School of Mechanical Engineering and Automation, Northeastern University, Shenyang, 110819, China

A R T I C L E I N F O

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A B S T R A C T

For the impact system in which a moving projectile transversely impacts against a taut fabric band, 1-D linearized model applies because of low-velocity, sufficient pretension, and the sizes of the objects. This projectile-to-band impact model can serve as the physical prototype of applications in engineering such as cable-membrane architectures and seat belts. In this fundamental work, the response properties under central and non-central impacts are investigated analytically from the viewpoint of wave propagations, while comparisons and verifications are made with finite element (FE) analysis. For a central impact after the first separation, band can catch up with the projectile such that a contact-impact state is re-established when m is in the small interval neighbouring $m = 1$. For a non-central impact, the projectile would be subjected to a combination of translation and rotation due to asymmetric wave propagations. From every certain instant, the projectile is subjected to an additional rotational acceleration (principal moment) with an abrupt or zero initial value in the anti-clockwise or clockwise direction. The swing amplitude of a small-j or a flat projectile is susceptible to significant fluctuations, and vice versa. The band with a rather large off-centre ratio for the impacted zone and a rather short length of the shorter segment would facilitate a larger accumulation of swing amplitude in a single direction soon after the impact. The linearized impact models proposed can be used to well describe the small-deflection responses for the system, based on 1-D wave propagations or the dependence of quasi-static band deflection on time if the impact duration is much longer than the double wave transit time for the band.

1. Introduction

Membrane structure, generally made of polymer, possesses light weight, thin shape, sufficient specific strength, and high flexibility. The need of its characteristics for withstanding the impact from external projectile has increased the use of membranes in applications such as civil engineering structures and soft armors [1]. For passive restraint system, the buffering process can lessen the stress harms to projectile while absorbing the impact energy, in contrast with stiff obstruction. For example, seat belt is responsible for intercepting human body to avoid sustaining injuries beyond the expected limit, and it can be reduced to an idealized rigid-to-string impact model that has 2-D planar motion. For a rigid-to-string impact system, it

* Corresponding author.
E-mail address: zhaoyifei13134@hotmail.com (Y. Zhao).

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Nomenclature

Latin

c	wave speed ($c = \sqrt{P/\rho}$)
D, d	front contact width
E, e	eccentricity
F, f	contact force (positive in compression)
G, g	energy
H	Heaviside function
J, j	moment of inertia
L	remaining length of band after excluding D
M, m	mass
P	initial longitudinal tension
R, r	moment imposed on projectile
T, t	time
V	impact velocity of projectile
W, w	transverse displacement of band under non-central impact
ΔW, Δw	transverse displacement difference of the two edges at the bottom of projectile
X, x	longitudinal coordinate
Y, y	transverse displacement of band under central impact

Greek

α	rotational angle
λ	aspect ratio (the ratio of W-directional height to D) of cuboid projectile
ρ	mass density per unit length

Superscripts

~	reference
←,→	left, right segment of band
•, ••	first, second derivative
I, R	incident, reflected wave
K, S	kinetic, strain energy

Subscripts

b	band
c	contact state
c-s	from contact state to separation state
max	maximum
p	projectile
rec	resumed contact
sep	separation
t	total energy of band and projectile

is interesting to gain insights into the interaction of the narrow band and projectile, especially in a general situation when the impacted zone of the band is not located at the mid-span.

With respect to an axially moving band having two free edges and two supported edges, the dynamic responses under large transverse (out-of-plane) deformation were investigated by different methods [2,3], particularly, Koivurova and Pramila [3] considered nonlinear effects such as the variations of tension and length between two roll-supports due to deformation. In terms of a narrow belt with pulley supports, using the perturbation method, Kim and Lee [4] analysed the influences of pretension, length, and elastic modulus on natural frequency, and the influence of belt types on the vibrational isolation under forced vibration. For a similar model but with a translating velocity, Shin et al. [5] presented the results of natural frequencies and mode shapes for in-plane vibration, considering aspect ratio and translating velocity. With respect to the responses under impacting loads, scholars directly used a rectangular pulse to denote the uniform force under wind impact in Ref. [1], and it was shown that both loading velocity and pretension have an influence on the dynamic response. Liu et al. [6] applied a concentrated impulse function with respect to time and central impacted point to represent the impacting load of