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Further investigation on the coupling between the reference and elastic displacements in flexible body dynamics

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A B S T R A C T

A new semi-analytical multibody system (MBS) model derived using general nonlinear flexible body dynamics formulation is developed in this investigation. The semi-analytical model can serve as a benchmark example for testing the implementation of the *finite element* (FE) *floating frame of reference* (FFR) formulation in commercial MBS software. In particular, this new model sheds light on the important concept of the *reference conditions* which is fundamental in the computer implementation of the FE/FFR formulation. In order to develop the new FE/FFR semi-analytical model, the reference motion of the flexible body is expressed in terms of specified motion trajectories, allowing for developing a set of second-order ordinary differential equations (ODE's) with time-varying coefficients. The FE/FFR *inertia shape integrals* are evaluated and used to define the generalized forces associated with the deformation coordinates. The simplifications that result from the use of the *mean-axis reference conditions* are highlighted and the significance of these simplifications on the solution accuracy when using more appropriate sets of reference conditions is discussed. It is shown that if a MBS computer algorithm restricts the choice of the reference conditions to one set, the use of other reference conditions can lead to a homogeneous ODE system with time-varying coefficients which has zero solution in the case of zero initial conditions and zero external forces regardless of the flexible body reference rotation.

1. Introduction

The *floating frame of reference* (FFR) formulation is widely used in commercial and research computer programs [1–10]. In the FFR formulation, the deformation of the body with respect to its coordinate system is defined uniquely using an appropriate set of reference conditions that eliminate the rigid body motion of the flexible body with respect to its coordinate system. Despite the wide use of the FFR formulation by automotive, aerospace, and construction and agricultural machine industries, existing benchmark examples fail to properly test the implementation of this formulation in commercial MBS software. The benchmark examples used in the literature are not designed to shed light on the generality of the implementation of this widely used formulation, and this in turn, has led to significant economic loss as the result of performing durability investigations based on wrong procedures. Therefore, the development of analytical and

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semi-analytical benchmark examples that test different aspects of the FFR implementation is necessary in order to ensure the credibility of the durability investigations and advance virtual prototyping to a level that allows reducing reliance on physical prototyping and experimental field-testing.

In the FFR formulation, the reference conditions must be carefully selected in order to obtain proper deformation-basis vectors consistent with the constraints imposed on the body motion. In some widely used commercial MBS computer programs, however, the choice of the reference conditions is restricted to one set which represents the free-free reference conditions, known as the *mean-axis reference conditions* [1, Pages 73, 74, 130, 132, 133]. The mean-axis reference conditions can lead to significant simplifications in the inertia coupling between the reference and elastic displacements. Nonetheless, these simplifications cannot be used with other sets of reference conditions that can be more appropriate for the definition of the body elastic coordinates.

In a recent analytical investigation [2], a simple static-force model was developed and used to shed light on important issues related to the durability analysis procedures adopted by the industry. A beam model subjected to a static load was developed and solved using FE commercial software. The FE solution was compared with the analytical solution obtained using the MBS/FFR formulation. It was shown that the use of the proper FFR reference conditions in MBS algorithms leads to the same solution obtained using the FE commercial software, while the use of improper reference conditions leads to a wrong solution. In particular, the use of the procedure recommended by some widely used commercial MBS software that restrict the choice of the reference conditions to the free-free reference conditions [1, Pages 73, 74, 130, 132, 133] led to a wrong solution with an error that exceeds 100% even for such a very simple example. The static-force example used in the literature [2] demonstrated clearly the problems encountered when FE-based durability studies are performed for MBS applications. This simple static-load example also shed light on the fundamental differences between the *FFR reference conditions*, the *structural mechanics boundary conditions*, the *sub-structuring interface conditions*, and the *MBS joint constraints* [2].

2. Background

The FFR reference conditions, which define a unique displacement field with respect to the body coordinate system, do not change the system topology and do not impose any kinematic constraints on the motion of the body reference. Therefore, these conditions are not the same as the structural mechanics boundary conditions, which define the system topology.

2.1. Credibility of the durability investigations

Misunderstanding of the fundamental differences between the FFR reference conditions, the structural mechanics boundary conditions, the sub-structuring interface conditions, and MBS joint constraints significantly compromises the quality of the durability investigations performed by the industry [2]. The error that exceeded 100% for a simple static-load example clearly demonstrated the following important facts that cannot be ignored: (1) Use of a conventional FE approach for durability investigations of MBS applications, including automotive, aerospace, and machine applications, leads to significant errors that cannot be ignored; (2) Structural mechanics boundary conditions cannot, in general, replace MBS joint constraints and cannot be used to develop durability analysis models that lead to credible results; (3) The conventional FE-based durability procedures used for MBS applications do not account properly for all the dynamic forces, and this procedure is equivalent to the linear theory of elasto-dynamics, which was introduced more than four decades ago and ignores the effect of the reference motion on the elastic deformation; and (4) The change in the system oscillation frequencies as the result of applying the actual MBS joints cannot be captured using a conventional FE analysis, and therefore, the resonance frequencies currently used in the FE-based durability investigations as the basis for the component designs can be misleading.

2.2. MBS joint constraints

It is also important to distinguish between different conditions (reference and sub-structuring-interface) and MBS joint constraints. The reference and sub-structuring interface conditions when used with the FFR formulation do not impose any constraints on the motion of the body reference, and therefore, these conditions do not change the system topology. MBS joint constraints, on the other hand, can impose restrictions on the motion of the body reference, eliminate reference degrees of freedom, and define the system topology. The FFR reference conditions are used to eliminate the rigid body motion of the body with respect to its reference, but does not restrict the motion of this reference. The sub-structuring interface conditions do not eliminate the rigid body motion with respect to the body reference, and therefore, they cannot serve as a substitute for the reference conditions. Nonetheless, there is no conflict when using sub-structuring techniques prior to imposing the reference conditions. The FFR reference conditions can still be imposed on the coordinates of the boundary nodes. Therefore, regardless of whether or not sub-structuring techniques are used, the use of the FFR reference conditions is necessary in order to define properly the body reference and its displacement field. With the advances in computer technology, however, sub-structuring techniques are not required in most MBS applications.