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Assessment of the apparent bending stiffness and damping of multilayer plates; modelling and experiment

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A B S T R A C T

In the context of aeronautics, automotive and construction applications, the design of light multilayer plates with optimized vibroacoustical damping and isolation performances remains a major industrial challenge and a hot topic of research. This paper focuses on the vibrational behavior of three-layered sandwich composite plates in a broad-band frequency range. Several aspects are studied through measurement techniques and analytical modelling of a steel/polymer/steel plate sandwich system. A contactless measurement of the velocity field of plates using a scanning laser vibrometer is performed, from which the equivalent single layer complex rigidity (apparent bending stiffness and apparent damping) in the mid/high frequency ranges is estimated. The results are combined with low/mid frequency estimations obtained with a high-resolution modal analysis method so that the frequency dependent equivalent Young's modulus and equivalent loss factor of the composite plate are identified for the whole [40 Hz-20 kHz] frequency band. The results are in very good agreement with an equivalent single layer analytical modelling based on wave propagation analysis (model of Guyader). The comparison with this model allows identifying the frequency dependent complex modulus of the polymer core layer through inverse resolution. Dynamical mechanical analysis measurements are also performed on the polymer layer alone and compared with the values obtained through the inverse method. Again, a good agreement between these two estimations over the broad-band frequency range demonstrates the validity of the approach.

1. Introduction

The combined high stiffness and light weight of sandwich composites make them increasingly used by today's transportation and construction industries for example. In this context, the design of light multilayer plates with optimized damping and isolation performances, for given frequency bands, remains a major industrial challenge and a hot topic of research. Thus, this article concerns the vibrational behavior of such lightweight composite plates in a broad-band frequency range.

Three analytical and four experimental vibroacoustics methods identifying the equivalent complex bending stiffness (or *flexural rigidity*) and equivalent loss factor of a three-layered plate are compared. These *equivalent* parameters, also known in the literature as *apparent* stiffness and *apparent* loss factor (see for example the studies from Nilsson [1] or Backström [2]), are of

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course frequency dependent. The purpose of this work is then to identify them up to the high-frequency domain where most of the experimental methods meet their limits in terms of precision and resolution (frequency or spatially). Several techniques are reported in the literature to handle such limitations, in particular wave number fitting approaches (see for example the work of Berthaut *et al.* [3] on ribbed structures and the one of Cherif *et al.* [4] on composite panels with honey comb core), image source methodologies (see for example the method of Cuenca *et al.* for estimating material properties of plates [5,6]) or the recently proposed Virtual Fields Method combined with Laser Doppler Vibrometry or optical deflectometry by Berry *et al.* [7,8]. Here we made the choice to compare four experimental approaches: traditional modal analysis [9], high-resolution modal analysis [10], CFAT methodology that uses a corrected finite differences scheme [11], and a wave correlation technique comprising an image source model that uses Hankel's functions [12].

Concerning the comparison with predictions, different approaches exist to model the vibrational behavior of multilayer plates (see for example a synthesis done by Carrera [13] or more recently the work of Shorter [14], Manconi and Mace [15], and Ghinet and Atalla [16], for thick composite laminated panels). Here three models are presented, compared and discussed in the first part of the manuscript - a) model of Guyader (traveling wave approach) [17,18] b) model of Ross, Kerwin and Ungar (strain energy) [19–21] c) Lamb waves model [22]. The plate under study (a steel/polymer/steel sandwich system) is then presented, and the experimental protocols and assessment procedures of the four experimental techniques considered are detailed in section 3. Using traditional modal analysis in the low frequency domain and finite-element model (FEM) calculations, the frequency dependent equivalent Young's modulus (derived from the equivalent bending stiffness) is identified up to 800 Hz. Then, a high-resolution modal analysis method allows identifying equivalent loss factor as function of frequency up to 2.5–3 kHz where modal overlap is high. Using two different methodologies (CFAT methodology and Hankel's functions image source model), contactless measurements of the velocity field extend the identification of the frequency dependent complex Young's modulus up to the very high frequency domain (20 kHz) and confirm both previous modal analyses estimations. Measurements and predictions are then compared and discussed in section 4. The article ends with the identification of the polymer core complex modulus that is finally compared with DMA measurements/extrapolations.

The chosen analytical and experimental methods do not limit this work to a benchmark study but rather aims at the discussion of measurement approaches that cover a wide frequency range. These measurements are validated with predictions, focusing on damping identification which is not straightforward in cases of high modal overlaps. Hence - extending a previous oral contribution [23] where only preliminary experimental results were given without theoretical discussions and fair comparisons - the principal novelties of this work can be summarized as follows:

- comparing for a given three-layered plate experimental modal analyses techniques (in the low/mid-frequency domains) to wave number fitting approaches (up to very high frequencies) with a focus on accurate damping characterization
- obtaining - over the whole [40 Hz–20 kHz] frequency range - *overlapping* experimental results in good agreement with an equivalent single layer plate model with frequency dependent *apparent* bending stiffness and damping
- using modelling predictions to identify through an inverse vibratory problem the missing mechanical properties of a given layer (here the polymeric core complex Young's modulus) and confirming this results with DMA measurements/extrapolations.

2. Equivalent single layer modelling

In this section three analytical modelling of multilayer are presented, compared and discussed on two examples of sandwich plates. Only main principles of the methods are recalled in this first section; more details on each of the techniques are given in appendices and corresponding cited references.

2.1. Model of Guyader

The first analytical approach considered is the model developed by Guyader et al. [17,18] which determines at a given frequency the equivalent complex flexural rigidity (or bending stiffness) $D^*_{\mathrm{eq}}(f) = D_{\mathrm{eq}}(f)\left(1 + \mathrm{j}\eta_{\mathrm{eq}}(f)\right)$ of a multilayer viscoelastic plate in the standard Love-Kirchhoff thin plate theory. The analytical method is based on the traveling wave approach applied to a simplified multilayer model. The bending, membrane and shear effects of each layers are considered and continuity conditions on displacement and shear stresses at each layer interface are used to obtain the equation of motion of the multilayer plate field expressed as a function of the first layer field. Hence the method determines for a given frequency of calculation the equivalent single layer plate complex bending stiffness under Love-Kirchhoff thin plate theory in order to exhibit the same transverse displacement that the multilayer plate. Details on the analytical derivation of this first approach considered here can be found in Refs. [17,18]. Note that some typing errors in the appendix of [17] have been corrected in the reimplementation of the model that was used in this paper.

Once this equivalent rigidity $D^*_{\mathrm{eq}}(f)$ at a given frequency f has been computed, the equivalent homogeneous material properties - density ρ_{eq}, Poisson's ratio ν_{eq}, Young's modulus $E_{\mathrm{eq}}(f)$ and loss factor $\eta_{\mathrm{eq}}(f)$ - of the multilayer are obtained as follow:

$$\rho_{\mathrm{eq}} = \frac{\sum_i h_i \rho_i}{\sum_i h_i} \tag{1}$$