Journal of Sound and Vibration 426 (2018) 258–277

Contents lists available at ScienceDirect

Journal of Sound and Vibration

journal homepage: www.elsevier.com/locate/jsvi

A modified variational method for nonlinear vibration analysis of rotating beams including Coriolis effects

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A R T I C L E I N F O

Article history:
Received 19 November 2017
Received in revised form 28 March 2018
Accepted 18 April 2018

Handling Editor: M.P. Cartmell

Keywords:
Rotating beams
Modified variational method
Nonlinear vibration
Coriolis effects

A B S T R A C T

This paper presents a general formulation for nonlinear vibration analysis of rotating beams. A modified variational method combined with a multi-segment partitioning technique is employed to derive the free and transient vibration behaviors of the rotating beams. The strain energy and kinetic energy functional are formulated based on the order truncation principle of the fully geometrically nonlinear beam theory. The Coriolis effects as well as nonlinear effects due to the coupling of bending-stretching, bending-twist and twist-stretching are taken into account. The present method relaxes the need to explicitly meet the requirements of the boundary conditions for the admissible functions, and allows the use of any linearly independent, complete basis functions as admissible functions for rotating beams. Moreover, the method is readily used to deal with the nonlinear transient vibration problems for rotating beams subjected to dynamic loads. The accuracy, convergence and efficiency of the proposed method are examined by numerical examples. The influences of Coriolis and centrifugal forces on the vibration behaviors of the beams with various hub radiuses and slenderness ratios and rotating at different angular velocities are also investigated.

1. Introduction

Rotating beams are widely used in many engineering applications such as helicopter rotors, aircraft propellers, wind turbines and robotic manipulators. The rotating beam is different from a non-rotating beam in having additional Coriolis effects and centrifugal force, which result in the considerable coupling of the structural modes in different directions [1–3]. When technological problems are related to their engineering application, it is always a challenge to have mechanical models balancing the opposite requests of being sufficiently accurate and efficient enough. In order to improve the computational efficiency, the free vibration behaviors of the rotating beams are often investigated by considering only the bending deformation, and the couplings among the elastic deformations of various directions are neglected [4–9]. When considering developing the mathematical model of the rotating beams, the governing equations could either be given in the strong or weak form. The strong form for the vibration problem of a rotating beam consists of nonlinear partial differential equations and associated boundary conditions [10,12]. The governing equations of the beam are general forth order nonlinear partial differential equations [15], therefore the motion equations generally cannot be solved in a closed form. In the weak form of

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https://doi.org/10.1016/j.jsv.2018.04.027

the governing equations, the motion equations can be given implicitly and written as a system of algebraic equations, which make the solution easier to be obtained than by solving the partial differential equations, directly. In the past years, several approximate techniques, such as Rayleigh-Ritz [12], Galerkin [17,18], finite element [7,10,13,14,31], power series [6,16], differential transform method [19–21], dynamic stiffness method [22,23] and differential quadrature method [24,25], have been proposed to solve the problems.

Among these approximate techniques mentioned above, the dynamic stiffness matrix method (DSM) is an efficient technique in the free vibration analysis of the beam. The DSM permits exact eigenvalue analysis, particularly when higher frequencies of results are required. This is true due to the reason that the properties obtained from the DSM are based on the closed form analytical solution of the partial differential equations. In this field, Banerjee and Kennedy [23] employed an exact dynamic stiffness method to analyze the in-plane free vibration of rotating beams including Coriolis effects, and the influences of rotational speed and hub radius on the natural frequencies and mode shapes are illustrated. Attarnejad and Shahba [22] derived the dynamic stiffness matrix for the general tapered beams by differential transform method. However, the application of the DSM method is limited to the special cases [26]. When the centrifugal force is not negligible, the centrifugal force cannot be assumed to be constant along the length of the beam. In this case, the governing partial differential equations of motion contain the variable coefficients, which make it impossible to achieve an analytical solution. Furthermore, when the couplings between various modes of elastic deformations are included, the solution of the partial differential equations may be too complex to obtain.

In recent years, power series method has also been widely used in the determination of the natural frequencies and mode shapes of rotating beams. In general, it is supposed that only one single segment is needed for power series solution to get the model frequencies and mode shapes for rotating beams [16]. In order to obtain the natural frequencies of a slender rotating beam at very high angular velocity, Huang et al. [6] proposed a practical power series method, which subdivides the rotating beam into several equal segments, and the accuracy as well as efficiency of the proposed method was confirmed by a series of numerical examples. However, the equations deduced by the boundary conditions in the power series method need to be homogeneous, which makes the method hard to be used to deal with the cases with special boundary conditions such as elastic boundary supports.

In most publications of the rotating beam, only the bending deformation is considered, whereas the Coriolis effects as well as nonlinear effects due to the coupling of bending-stretching, bending-twist and twist-stretching have always been neglected. However, those effects may significantly affect the dynamic behavior of rotating beams when the aeroelasticity has been taken into consideration [10–12]. Consequently, in this paper, the Coriolis effects and coupling among flexural, torsional and extensional deformation are introduced by using the order truncation principle [10] of the fully geometrically nonlinear beam theory [13,14]. Moreover, if studies focus on the fluid-structure interaction problems, the accurate description of the nonlinear transient dynamic behaviors for the rotating beam would also be very necessary, which is rarely reported. Thus, in this work, a modified variational method is proposed to accurately predict both the free and transient nonlinear vibration performance of the rotating beam.

In the supposed method, the rotating beam is subdivided into several beam segments. To achieve satisfactory accuracy, the displacement constraints and the stress constraints on the common boundaries of adjacent beam segments are considered. The boundary and interface conditions of each beam segment are included by the form of boundary potentials, and the governing equations of motion are deduced implicitly. The free vibration performances of the rotating beam are obtained by linearizing the nonlinear governing equations at the equilibrium position with Jacobian method. To accurately get the transient vibration performances of the rotating beam, the generalized-α time integration method [29] is employed. In this study, the additional Coriolis effects and centrifugal force have been taken into account. The accuracy and convergence of the proposed method are examined by numerical examples. The influences of Coriolis and centrifugal forces on the vibration behaviors of beams with various hub radiuses and slenderness ratios and rotating at different angular velocities are also presented.

2. Mathematical formulation

2.1. Problem description

Most rotating machineries such as helicopter rotors, aircraft propellers, wind turbines and robotic manipulators can be simplified into the rotating beam model shown in Fig. 1. The uniform rotating beam is mounted on the periphery of a rigid hub of the beam and rotates about the center of the hub. The cross-section of beam can be arbitrary. To accurately predict the vibration characteristics of the rotating beam, the Coriolis effects as well as nonlinear effects due to the coupling of bending-stretching, bending-twist and twist-stretching are considered in this paper. In order to establish the corresponding mathematical model, the length of the beam is L, the radius of the rigid hub is R, and the rotating speed of the beam is Ω in this paper.

The approaches to modelling the vibration of a rotating beam can be divided into the strong and weak form. In previous articles, some strong form methods, such as Galerkin and dynamic stiffness method, have been proven to be effective to establish a simplified rotating beam model. However, when the coupling effects among bending, stretching and torsional motions need to be taken into account, it will become very difficult to establish and solve the model of the rotating beam by strong-form methods. The difficulties are mainly manifested in two aspects: (1) it is very difficult to get the explicitly form of