Journal of Sound and Vibration 426 (2018) 111–128

Contents lists available at ScienceDirect

Journal of Sound and Vibration

journal homepage: www.elsevier.com/locate/jsvi

Dynamic condensation of non-classically damped structures using the method of Maclaurin expansion of the frequency response function in Laplace domain

Armin Esmaeilzad [a, *], Karen Khanlari [b]

[a] Central Tehran Branch, Azad University, Iran
[b] Faculty of Technical and Engineering, Central Tehran Branch, Islamic Azad University (IAUCTB), Iran

ARTICLE INFO

Article history:
Received 21 November 2016
Received in revised form 20 March 2018
Accepted 10 April 2018

Keywords:
Dynamic condensation
Dynamic reduction
Degrees of freedom
Non-classical damping
System identification
Damage detection

ABSTRACT

As the number of degrees of freedom (DOFs) in structural dynamic problems becomes larger, the analyzing complexity and CPU usage of computers increase drastically. Condensation (or reduction) method is an efficient technique to reduce the size of the full model or the dimension of the structural matrices by eliminating the unimportant DOFs. After the first presentation of condensation method by Guyan in 1965 for undamped structures, which ignores the dynamic effects of the mass term, various forms of dynamic condensation methods were presented to overcome this issue. Moreover, researchers have tried to expand the dynamic condensation method to non-classically damped structures. Dynamic reduction of such systems is far more complicated than undamped systems. The proposed non-iterative method in this paper is introduced as 'Maclaurin Expansion of the frequency response function in Laplace Domain' (MELD) applied for dynamic reduction of non-classically damped structures. The present approach is implemented in four numerical examples of 2D bending-shear-axial frames with various numbers of stories and spans and also a floating raft isolation system. The results of natural frequencies and dynamic responses of models are compared with each other before and after the dynamic reduction. It is shown that the result accuracy has acceptable convergence in both cases. In addition, it is indicated that the result of the proposed method is more accurate than the results of some other existing condensation methods.

1. Introduction

One of the major issues in the structural dynamic problems is the determination of eigenvalues, eigenvectors and dynamic responses. In order to accurately evaluate the structural dynamic characteristics using finite element method, the number of degrees of freedom (DOFs) has become larger and larger. Thus, solving the problems with a large number of DOFs is not unusual [1]. Despite the fact that super-advanced computers are able to solve the problems with more than a million DOFs, still it is very essential in such problems to reduce the computation time and overcome the memory limitations by reducing the complexity of FE models [2]. Reducing the structural size is one of the useful methods for

* Corresponding author.
E-mail address: armin_es2002@yahoo.com (A. Esmaeilzad).

https://doi.org/10.1016/j.jsv.2018.04.016

reducing the computational time. So far, various methods of model reduction have been proposed by researchers either by (1) 'Reduced order method,' that relies on truncating the higher frequencies or (2) 'Condensation' method in which the reduced matrices are generally constructed via a transformation matrix [3]. In the second case, the DOFs are partitioned to identify those that should be retained (master DOFs) and ones that are not important and can be eliminated (slave DOFs). Dynamic condensation is a highly efficient method for model reduction and was first applied in large finite element models to compute the natural frequencies and mode shapes faster. Since eliminating the slave DOFs make a smaller model than the full model, the computational efforts and time analysis are decreased. In recent years, model reduction has also been widely used in test-analysis model correlation, vibration control, structural dynamic optimization, dynamic modeling [4], system identification and damage detection. All the related researches have benefitted from the 'Static condensation method' for undamped structures proposed firstly by Robert Guyan [5] and Bruce Irons [6] in 1965. In this method, the mass term is completely ignored. Many types of algorithms have been developed in order to achieve dynamic condensation for undamped or proportionally damped models such as those proposed by Kidder [7], Paz [8], O'Callahan [9], Flanigan [10], Bouhaddi [11], Suarez and Singh [12], Flippen [13–15], Friswel [16], Qu [17] and Khanlari [18]. In proportionally damped systems proposed by Caughey and O'Kelly [19], the natural modes of system vibration are all real-valued and identical to undamped conditions. Structures satisfying this condition are said to be classically damped [20]. In particular, a well-known strategy for obtaining a dynamic condensation to the essential DOFs of structures is the dynamic stiffness approach in combination with the Wittrick and William algorithm for undamped structures [36,37]. Nowadays, some researchers still study the method of dynamic condensation on undamped or classically damped structures [27]. However, in many kinds of dynamic structures, the damping could not be ignored or modeled proportionally. The structures which their damping matrices are not a linear combination of mass and stiffness matrices, are called non-classically damped structures. They generally have complex valued natural modes. This is the case for structures made of different materials with different damping properties in different parts, structures equipped with passive and active dampers and controllers, structures with tuned mass or tuned liquid dampers, structures made from intelligent or viscoelastic materials and those affected by issues related to the soil-structure interacting systems. Qu and Cheng [21], Rivera et al. [22] and Qu and Selvam [2,4,23] are among the researchers who have worked on the dynamic condensation method for non-classically damped structures. In most of these methods, the aim is to reduce the structure size by an iterative procedure. The extension of the dynamic stiffness method in conjunction with a general eigen-solution strategy for non-classically damped structure represents a very important research task. Moreover, in both static and dynamic problems, if there is the possibility of dividing the whole structure into substructures, then it can be solved more readily with limited memory [24–26]. However the scope of this paper is not about substructuring or domain decomposition. The computational effort of system analysis is approximately proportional to the cubic of the size of the structure. Thus, if the size of the structure is reduced, the computational work for dynamic analysis could be reduced drastically [32]. From the total of previous studies, it is concluded that the amount of research provided for condensation of non-classically damped structures is not enough. In addition, these papers only have focused on the determination of natural frequencies and have paid less attention to determination of the dynamic responses. Determining the dynamic responses of the structure is one of the most important parts of the dynamic analysis. In the present study, a non-iterative dynamic condensation method is proposed for non-classically damped structures to increase the accuracy of the former methods and also examine the dynamic responses of displacement, velocity and acceleration of two-dimensional (2D) frame structures under earthquake acceleration record.

2. Proposed methodology

2.1. Laplace transform

The dynamic equation for non-classically damped structures can be expressed in matrix form as:

$$\mathbf{M}\ddot{\mathbf{x}}(\mathrm{t}) + \mathbf{C}\dot{\mathbf{x}}(\mathrm{t}) + \mathbf{K}\mathbf{x}(\mathrm{t}) = \mathbf{f}(\mathrm{t}) \tag{1}$$

where

$\mathbf{M}$ = mass matrix with the size of $(NDOF \times NDOF)$
$\mathbf{C}$ = damping matrix with the size of $(NDOF \times NDOF)$
$\mathbf{K}$ = stiffness matrix with the size of $(NDOF \times NDOF)$
$NDOF$ = number of total DOFs
$\ddot{\mathbf{x}}(\mathrm{t})$ = acceleration response matrix
$\dot{\mathbf{x}}(\mathrm{t})$ = velocity response matrix
$\mathbf{x}(\mathrm{t})$ = displacement response matrix
$\mathbf{f}(\mathrm{t})$ = external force matrix

The non-classically damped matrix which has no linear relationship with mass and stiffness matrices, is found by the following method [28,29]