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On the use of a roving body with rotary inertia to locate cracks in beams

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A R T I C L E I N F O

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A B S T R A C T

Identifying cracks and damages in structures using measured vibrational characteristics has received considerable attention in the past few decades. The possibility of using frequency changes due to the application of a mass appended to the structure has also been considered. In this paper an analytical proof to show that the natural frequencies of a cracked beam with a roving body possessing mass and rotary inertia will generally change abruptly as the body passes over a crack, provided that the crack permits differential flexural rotations, is presented. A novel explicit closed form solution of the governing equation of an Euler-Bernoulli beam with a roving body possessing mass and rotary inertia, in the presence of multiple cracks is also proposed. The presented exact solution is used to conduct a parametric analysis of cracked beams. Numerical results for natural frequencies are provided and a procedure to exploit the occurrence of frequency shifts to detect and locate each crack, without having to perform any additional calculation, is described.

1. Introduction

Structural integrity of engineering constructions and structures is affected by the normal process of deterioration which can compromise their safety and serviceability. In the past few decades many efforts have been taken by engineers to detect and localize damage in its early stage using vibrational measurements as is implicit in the comprehensive literature review on structural health monitoring and damage identification presented by Doebling et al. [1].

Most of the vibration measurements came from acceleration data that have to be processed in order to extract modal parameters, such as the natural frequencies, to characterise the dynamics of the structure. In the words of Doebling et al. [2]: *"The vibration based identification methods rely on the fact that damage generates singularities in the modal parameters"*.

As mentioned in Ref. [3] only a significant damage would cause a measurable change when the damage detection is carried out by analysing the changes in the natural frequencies and the damage effect could be veiled by environmental changes and experimental uncertainties. To overcome the problem of sensitivity of the changes in natural frequencies, Pandey et al. [4] proposed to use the mode shapes to capture the discontinuity produced by the damage. Mode shapes have also been used to reconstruct position and severity of multiple cracks by proposing a sequential identification

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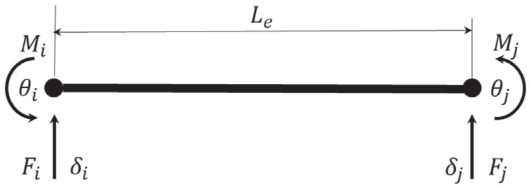

Fig. 1. Sign convention for beam elements.

procedure [5]. However, compared with the natural frequency measurements, the identification of mode shapes requires additional experimental and mathematical resources, and the effect of small damage on the modes is normally of the order of experimental noise and environmental effects, making the identification problem a complicated task [3]. It is worth noting that several identification procedures assume the number of concentrated cracks as a known variable, but in real applications this is not the case. Thus there is a need to develop a reliable identification procedure that is able to detect the exact number and location of the cracked cross sections which is the motivation behind the work presented in this paper.

In order to easily distinguish the changes induced by damage, Zhong and Oyadiji [6] and Solís et al. [3] use wavelet coefficients as an indicator of damage when they are applied to the differences in mode shapes between the undamaged and the damaged beams. It may be noted that Zhong and Oyadiji [6] and Bahador and Oyadiji [7] performed the modal analysis for different positions of a non-structural mass attached to the structure in order to make the damage detection method more robust and sensitive. The roving mass method proposed by Zhong and Oyadiji in Ref. [8] involves locating the mass at different positions at each round of test, but during the vibration measurement the mass is not allowed to move axially, so its velocity effects need not be considered.

In this paper, besides the translational inertia (mass) of the roving body, the effect of its rotational inertia is also taken into account. It should be noted that the rotary inertia due to the self mass of the beam is not considered in this work. That is the beam is an Euler-Bernoulli beam and not a Timoshenko beam. The idea of using the rotary inertia of the roving body is to exploit the fact that, since the damage produces a rotational discontinuity at the crack location, it is expected that the rotary inertia would introduce a jump in the frequency as the roving body crosses the crack position. The theoretical proof of the latter statement and the influence of each of the parameters in the determinantal equation will be discerned following the derivations explained in Ref. [9] using the Dynamic Stiffness Matrix. A procedure, based on the detection of the frequency jumps, that generally enables identification of the number of cracks present in a beam and their location is proposed. The most significant feature of this procedure is that it does not require any knowledge of the dynamic properties of the corresponding healthy beam. Furthermore, a closed form integration of the governing free vibration equations of a multi-cracked beam, in the presence of a roving body with both translational and rotary inertia, is derived to numerically analyse the performance of the proposed identification method aiming at providing a first investigation versus its practical applicability. The results have also been verified by comparing with results obtained using the Finite Element Method.

2. Frequency shift derivation using dynamic stiffness matrix (DSM)

According to the rotational stiffness model, a cracked beam is split at the crack position into two segments which are connected by a hinge allowing a discontinuity in the slope with a partial rotational restraint modelled by a rotational spring of stiffness K which is related to the severity of the damage [10]. The connecting spring is also referred to as a torsional spring but it should be noted that it restrains the relative flexural rotation only and not the angle of twist. When considering only transverse and angular displacements using the dynamic stiffness approach there are two degrees of freedom for vibration in one plane at each end of any beam element of length L_e with the following assumed sign convention, Fig. 1.

The exact dynamic stiffness equations for each beam element with Young's modulus E, second moment of area I and mass per unit length m, vibrating at frequency ω is given in Ref. [11] as follows:

$$\begin{Bmatrix} F_i \\ M_i \\ F_j \\ M_j \end{Bmatrix} = \begin{bmatrix} a_e & b_e & -d_e & e_e \\ b_e & c_e & -\varepsilon_e & f_e \\ -d_e & -\varepsilon_e & \alpha_e & -\beta_e \\ e_e & f_e & -\beta_e & \gamma_e \end{bmatrix} \begin{Bmatrix} \delta_i \\ \theta_i \\ \delta_j \\ \theta_j \end{Bmatrix} \tag{1}$$

where, for a uniform member: