The modelling of the flow-induced vibrations of periodic flat and axial-symmetric structures with a wave-based method

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ABSTRACT

The stochastic response of periodic flat and axial-symmetric structures, subjected to random and spatially-correlated loads, is here analysed through an approach based on the combination of a wave finite element and a transfer matrix method. Although giving a lower computational cost, the present approach keeps the same accuracy of classic finite element methods. When dealing with homogeneous structures, the accuracy is also extended to higher frequencies, without increasing the time of calculation. Depending on the complexity of the structure and the frequency range, the computational cost can be reduced more than two orders of magnitude. The presented methodology is validated both for simple and complex structural shapes, under deterministic and random loads.

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1. Introduction

The vibrations, induced by a convective or an acoustic field exciting an elastic structure, arise strong interest in many engineering areas. These induced vibrations can even damage internal devices and payloads in the case of a space-launcher in the lift-off phase. The resulting radiated noise is, instead, a problem in the case of transport means in which the internal acoustic comfort can have a great importance: an aircraft fuselage, a train or vehicle cabin. As a result, the random vibrational and noise levels on the structures must be predicted in the design phase, in order to fix eventual issues before operating the system.

The response of fluid loaded structures can be estimated from data on wall pressure fields (WPF) and their correlation functions. The first characterisation of the wall pressure fluctuations has been carried out by Corcos [1], on experimental measurements. This model, which does not take into account the position of each point on the structure, considering just the relative distances, is one of the simplest and most used in literature. Specifically, the cross spectral density of the pressure fluctuations, through the assumption of separation of variables, is function of the relative distances among points, the convection velocity and the frequency.

Many other models have been proposed to improve the accuracy of the Corcos model in the estimation of the WPFs (Wall Pressure Fluctuations) in the sub-convective domain, which is the wavenumber domain comprised between the acoustic and the convective wavenumber. Some of these models are directly linked to the Corcos one [2,3], on the contrary, others do not

take into account the separation of the stream-wise and cross-wise coherence [4,5].

Most of the literature, regarding flow-induced vibrations, is limited to very simple cases, such as a flat rectangular simply-supported panel subjected to a specific type of fluid turbulence model. Even if this is a useful test-case, it is not industrially relevant and only few more complex applications are available. The predictive methodologies actually available in literature are mainly modal-based and energy-based [6–10].

Among the modal approaches, there is the full finite element approach, which makes use of a discretization of the structural operator in finite elements, whose size is, generally, strongly affected by the fluid loading wavelength at the design frequency of analysis, [10]. This is, for most of applications, lower than the structural one, increasing, thus, the size of the problem in terms of degrees of freedom (DoF). Some authors proposed different methods to reduce the cost issues associated to this approach. For example, a method of load approximation through equivalent deterministic forces, analysing the eigenvalues distributions of the load matrix for a general convective load distribution, has been proposed in Ref. [11]. Two frequency ranges are identified in which the convective load can be approximated through a purely coherent or purely incoherent equivalent load, proposing thus, a pseudo equivalent deterministic excitation method as an upgrade of the classic pseudo excitation method [11]. This approach proved to work very well in the *low* or *high* frequencies since the behaviour of the eigenvalues of the load matrix is easily predictable. In the *mid* frequencies, some approximations have to be made and the accuracy lowers [12].

Another possibility is to use scaled models or similitudes to reproduce the response of smaller systems, which have a reduced computational cost for given wavelength to describe, assuming fixed mesh sizes. In these cases, the main issues, are associated to the application of scaling laws to rebuild the response of the original system [8,12].

On the other hand, moving to high-frequencies, energy-based approaches can be used. Among the methods proposed in literature, the most effective is surely the equivalent rain-on-the-roof method, [7]. This approach considers the fact that even spatially-correlated fluctuations, when moving to high-frequencies, head towards uncorrelated models. Then, it is possible to evaluate an equivalent uncorrelated load which approximates the real load correlation function. This gives huge advantages to the calculation of the joint acceptance integral, [7].

Most of these techniques have limits associated to the accuracy and the computational cost in certain frequency ranges, especially in the medium one.

This paper deals with the modelling of the flow-induced vibrations of periodic and axial-symmetric structures when random excitations act on the external surface of the structure by means of the pressure fluctuations, inducing vibrations and radiated noise inside the structural cavities. A wave-based methodology, within a transfer matrix framework, is proposed, in order to reduce the computational cost. On the other hand, for a given computational cost, the frequency range of accuracy is extended overcoming the actual state of the art, in particular for axial-symmetric structures.

The work is structured as follows: Section 2 presents a theoretical background; Section 3 proposes and explains the methodologies and the new formulation for axial-symmetric structures. The final section presents the results for all the test cases.

2. Theoretical background

2.1. The fluid: Corcos model

The coherence model for the wall pressure fluctuations proposed by Corcos can be expressed through the product of two separate functions in the stream-wise and cross-wise directions, respectively:

$$X_{pp}(\xi, \omega) = S_p(\omega)\Gamma(\xi_x, \xi_y, \omega) \tag{1}$$

where

$$\Gamma(\xi_x, \xi_y, \omega) = e^{-\alpha_x|\omega\xi_x/U_c|}e^{-\alpha_y|\omega\xi_y/U_c|}e^{i|\omega\xi_x/U_c|} \tag{2}$$

U_c is the convective flow speed, S_p is the single-point auto spectral density of the wall pressure distribution, α_x and α_y are the correlation coefficients and ξ_x and ξ_y are the relative distances for the stream-wise and cross-wise directions respectively (Fig. 1). Some assumptions are present in this model as the separation of variables, the exponential form of the functions, the dependency on the distances instead of the point locations and the stream-wise harmonic variation accounted. It has to be underlined, also, that the correlation coefficients, α_x and α_y, are to be determined from wind tunnel gallery measurements of the spatial coherence of the wall pressure fluctuations, which is, of course, a limit to its applicability.

However, this model is extensively used in literature as a predictive model and it is used in this paper too. This is due to the possibility to use a range of universal values for the correlation coefficients if the assumptions of no-gradients effects and fully developed turbulent boundary layer (TBL) are assumed. In any case, more than other models, the Corcos one is the best choice for simplicity versus predictive capabilities.