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The reduced space Sequential Quadratic Programming (SQP) method for calculating the worst resonance response of nonlinear systems

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ABSTRACT

A coupled approach combining the reduced space Sequential Quadratic Programming (SQP) method with the harmonic balance condensation technique for finding the worst resonance response is developed. The nonlinear equality constraints of the optimization problem are imposed on the condensed harmonic balance equations. Making use of the null space decomposition technique, the original optimization formulation in the full space is mathematically simplified, and solved in the reduced space by means of the reduced SQP method. The transformation matrix that maps the full space to the null space of the constrained optimization problem is constructed via the coordinate basis scheme. The removal of the nonlinear equality constraints is accomplished, resulting in a simple optimization problem subject to bound constraints. Moreover, second order correction technique is introduced to overcome Maratos effect. The combination application of the reduced SQP method and condensation technique permits a large reduction of the computational cost. Finally, the effectiveness and applicability of the proposed methodology is demonstrated by two numerical examples.

1. Introduction

Predicting correctly the nonlinear resonance behaviors of nonlinear systems is a challenging task for structure design [1–3]. For example, the resonance peak of nonlinear systems can be found with Den Hartog's equal-peak method and the optimal nonlinear coefficient of the Nonlinear Tuned Vibration Absorber(NLTVA) is determined numerically [4,5]. A more detailed study about the performance and robustness of the NLTVA is numerically elaborated in Ref. [6] and experimentally validated in Ref. [7]. However, the existence of isolated solution is an important hindrance for finding the resonance peak. Some methods are proposed for solving this tough problem. For example, through combining the harmonic balance technique with a continuation method, Detroux et al. [8]identified the isolated resonance curve in the forced response of a satellite structure. In Ref. [9], a harmonic balance approach is developed for detecting and tracing simple bifurcations and imperfect bifurcation with isolated branches.

The harmonic balance method has been demonstrated successfully in many researches. For example, the higher order harmonic balance method for the continuation of periodic solution and quasi-periodic motions is developed in Refs. [10,11].

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Joannin et al. [12,13] presented a nonlinear component mode synthesis method for analyzing the steady-state forced response of nonlinear and dissipative structures. Based on linear system analysis with specifically chosen boundary conditions on the contact interface, the adaptive microslip projection reduction basis is proposed in Ref. [14]. In Refs. [15,16], the axial and transversal dynamic behavior of the "Harmony" assembly has been studied both experimentally and numerically. In order to improve the convergence and computational efficiency of the Newton–Raphson algorithm, the analytical Jacobian matrix of the harmonic balance algebraic system for turbine blade with 3D shroud contact model is derived in Ref. [17]. It should be noted that the mentioned methods are all resorted to root-finding approach, in which all involved variables are solved by Newton-like methods.

Several researches have showed that the harmonic balance method can successfully be used in conjunction with gradient based optimizers to find the periodic solution [18,19]. The gradient based method has well behavior on computing speed and it has a great advantage on solving large-scale optimization problems. To implement the gradient based optimization algorithm, the gradients of the objective and constraint functions with respect to the optimization variables are required. Several approaches are available for efficient gradient computation. One choice is using the finite difference method. However, it is not efficient for large scale problems involving a large number of variables. If the dimension of the optimization space is rather large, the adjoint method [20–22] is preferred.

Two applicable methods are proposed for solving nonlinear optimization problem: the sequential approach (or Nested Analysis and Design) and the simultaneous optimization method (or Simultaneous Analysis and Design) [23,24]. The sequential method can be regarded as two loops. The outer loop is based on the optimization algorithm for solving the optimization problem, while the inner loop is dedicated to solve the governing equations. The sequential approach reduces the computational complexity for solving the entire problem. However, the sequential method suffers from severe drawbacks. Since many samples have to be tested, the sequential method may be very expensive from a computational point of view. In addition, the conventional sequential method solves the physical problem independently during the optimization process, where the effects of interaction between the outer loop and the inner loop are neglected. Solving the physical problems in an isolated manner often leads to a sub-optimal solution.

An attractive alternative to the sequential optimization approach is offered by simultaneous optimization. The core of the simultaneous method is to integrate physical problem solving in the optimization process. In comparison to the sequential optimization method, the simultaneous procedure leads to a significantly reduced computational cost. It has been demonstrated in literatures that the application of simultaneous optimization method for solving the structural dynamics problems is promising. In an earlier work [25], a novel strategy is presented to merge the harmonic balance method and the stability analysis technique in the constrained optimization framework. Moreover, Liao [26] investigated the Duffing oscillator with two kinds of fractional-order derivative terms by using the improved version of the constrained optimization harmonic balance method. In order to locate the isolated solutions in the global parameter space, a hybrid method which incorporates the advantages of global exploration and local exploitation is developed in Ref. [27].

It should be noted that the computational complexity is significantly increased when the harmonic balance equations are included in the gradient optimization due to the much larger number of optimization variables and constraints. A major drawback of the constrained optimization harmonic balance method is that the computational costs grow as the number of optimization variables increases, which motivates the search for new approaches with considerably lower computational cost.

The reduced Sequential Quadratic Programming algorithm [28–32] is efficient for large scale nonlinear problems with low degree of freedom. The reduced approach is of particular interest if the size of the design space is much smaller than the size of the state space. Biegler et al. [33,34] proposed a reduced Hessian method for large-scale constrained optimization. Furthermore, Leineweber et al. [35] performed the reduced space SQP algorithm to solve large-scale chemical process multistage optimization problem using multiple shooting principle.

In view of the problems mentioned above, the introduction of a suitable reduced space numerical scheme is instrumental to reduce both the storage requirements and the computational complexity. In order to achieve these goals in this contribution, the reduced SQP technique is implemented for solving the nonlinear constrained optimization problem resulted from the application of the harmonic balance method to solve the worst resonance response problem. To the best knowledge of the authors, it is the first time that the reduced SQP algorithm is coupled with the harmonic balance method for solving the resonance problem of nonlinear systems.

The layout of this paper proceeds as follows: Section 2 is devoted to proper optimization formulation of the worst case resonance response problem. The condensation representation of harmonic balance equations for the nonlinear equality constraints as well as the formulas to perform the sensitivity analysis is derived. The key principles and implementation characters of the reduced SQP are presented to solve the nonlinear constrained optimization problem. Two numerical examples are illustrated to demonstrate the efficiency and accuracy of the proposed method in Section 3. The calculations are initially performed for simple cases to validate the method. Subsequently, more complex scenarios are analyzed to present the full potential of the method. Finally, concluding remarks are gathered in Section 4.

2. The proposed method

The mathematical formulations and implementation of the proposed method are described in this section. The nonlinear equality constraint optimization problem with condensation harmonic balance equations is formulated, and the sensitivities