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Singular boundary method for wave propagation analysis in periodic structures

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ABSTRACT

A strong-form boundary collocation method, the singular boundary method (SBM), is developed in this paper for the wave propagation analysis at low and moderate wavenumbers in periodic structures. The SBM is of several advantages including mathematically simple, easy-to-program, meshless with the application of the concept of origin intensity factors in order to eliminate the singularity of the fundamental solutions and avoid the numerical evaluation of the singular integrals in the boundary element method. Due to the periodic behaviors of the structures, the SBM coefficient matrix can be represented as a block Toeplitz matrix. By employing three different fast Toeplitz-matrix solvers, the computational time and storage requirements are significantly reduced in the proposed SBM analysis. To demonstrate the effectiveness of the proposed SBM formulation for wave propagation analysis in periodic structures, several benchmark examples are presented and discussed The proposed SBM results are compared with the analytical solutions, the reference results and the COMSOL software.

1. Introduction

The control of wave propagation in periodic structures [1–3] is of great importance in the design and manufacture of modern acoustic and optical devices, such as photonic crystals, photovoltaic devices and metamaterials [4,5]. To improve the performance of these devices, various numerical methods have been proposed to simulate and manipulate wave propagation in periodic structures, such as the transfer matrix method [6], multiple scattering theory method [7], finite element method (FEM) [8], boundary element method (BEM) [9], plane wave expansion method (PWEM) [10], method of fundamental solutions (MFS) [11], to mention just a few of them.

Among the above-mentioned numerical methods, the transfer matrix method is mainly implemented to 1D periodic structures. The multiple scattering theory method is limited to solving 3D periodic structures with special scatterers (sphere or cylinder). The domain-discretization FEM requires the additional treatments to carefully deal with the exterior domain of the periodic structures. The BEM [12] requires only boundary discretization, and introduces the fundamental solution as its basis function to satisfy the governing equation and the Sommerfeld radiation condition at infinity in advance. However, it is a

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sophisticated mathematical and time-consuming issue for numerical integration over the singularities in the BEM. Besides, both the PWEM and MFS can avoid these troublesome issues appeared in the BEM due to the use of the nonsingular plane wave basis function or the placement of the source nodes away from the real physical boundary. However, both the PWEM and MFS provide the highly ill-conditioning resultant matrices with moderate node number, which may jeopardize the accuracy of their numerical solutions. In addition, the PWEM has a relatively slow convergence rate. In the MFS, the determination of the efficient placement of the source nodes is vital for numerical accuracy and reliability, and it requires an additional computational cost.

Based on the merits and demerits of the above-mentioned numerical methods, a meshless boundary collocation method, the singular boundary method (SBM), has been proposed by Chen et al. in 2009. The SBM [13,14] employs the fundamental solutions as the basis functions, and introduces the concept of "origin intensity factors" (source intensity factors) to take the place of the singularities encountered in the fundamental solutions at origin. It inherits the merits from the BEM and the MFS, and avoids the numerical computation of the singular integrals in the BEM, and circumvents the troublesome placement of the source nodes in the MFS. In addition, unlike the other boundary collocation methods (MFS and PWEM), the resultant matrix obtained by the SBM has the reasonable condition number as same as the BEM.

It should be mentioned that the determination of the origin intensity factors (OIFs) is one of the key issues in the SBM implementation. So far, there are four approaches [15,16] proposed to determine the OIFs of both the fundamental solutions and their derivatives. The efficiency and accuracy of the SBM have been verified for the potential problems, acoustic and elastic waves, and water waves with arbitrary complex-shaped geometries. Table 1 lists the conclusion for the comparisons between these four approaches in the SBM, where SLE denotes the system of linear equations. In the table, the more "*" it has, the better it is.

Besides, the SBM resultant matrix is very large and dense similar to the BEM. It is necessary to combine the SBM with the additional techniques to alleviate the computational and storage requirements for large-scale applications, such as the fast multipole methods [17,18], the adaptive cross approximation [19], the Fast Fourier Transform (FFT) [20,21] and the pre-corrected Fast Fourier Transform (pFFT) [22,23]. It has been shown that the SBM formulation for the periodic structures leads to a Toeplitz-type matrix similar to the BEM [24]. Therefore, several efficient and fast algorithms can be used to accelerate the computation of the SBM with the special structure of Toeplitz matrix (constant elements along each diagonal). Levinson [25] derived an $O(n^2)$ algorithm for a $n \times n$ Toeplitz matrix. Chandrasekaran and Sayed [26] proposed a stable fast solver for non-symmetric Toeplitz-type matrices. Chan [27] introduced the iterative solvers for Toeplitz matrices. Ferreira and Dominguez [28] proposed an $O(n \log n)$ algorithm, which extends a Toeplitz matrix to a circulant matrix by adding more equations, and implements the simple iterative algorithm in conjunction with the Fast Fourier Transform (FFT) to obtain the solution of Toeplitz matrix. Karimi et al. [29] introduced the algorithm developed by Ferreira and Dominguez into the BEM for acoustic wave analysis in periodic structures.

In this study, the SBM in conjunction with Fast Toeplitz-type Matrix Solvers (FTMS) is introduced to the wave propagation analysis at low and moderate wavenumbers in periodic structures. In the SBM implementation, the Approach4, the empirical formula coupled with the subtracting and adding-back technique [15], is used to determine the origin intensity factors. In addition, three fast Toeplitz-type matrix solvers are implemented according to the types of the SBM resultant matrix. The paper is divided into the following sections. In Section 2, a detailed numerical implementation of the proposed SBM-FTMS model is described. Section 3 provides some numerical examples to demonstrate the effectiveness of the proposed method. Finally, in Section 4 some conclusions are drawn from the present analysis.

2. Methodology

2.1. Time-harmonic wave propagation model

Consider the time-harmonic wave propagation in a homogeneous and isotropic medium D exterior to the closed bounded curve Γ of the periodic structures, which is described by the Helmholtz equation

$$\nabla^2 u(x) + k^2 u(x) = 0, \quad x \in D, \tag{1}$$

subjected to the boundary conditions

$$u(x) = \overline{u} \quad x \in \Gamma_D, \tag{2a}$$

Table 1
The comparisons between these four approaches in the SBM.

	Approach1	Approach2	Approach3	Approach4
Accuracy	**	****	*	***
Stability	*	***	****	****
Easy to use	*	**	***	****
Inconvenience	Inner sample node, Solve SLE twice	Boundary sample node, Solve SLE twice	Lowest accuracy, Numerical Integral	/