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Selection of regularization parameter for l_1-regularized damage detection

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ABSTRACT

The l_1 regularization technique has been developed for structural health monitoring and damage detection through employing the sparsity condition of structural damage. The regularization parameter, which controls the trade-off between data fidelity and solution size of the regularization problem, exerts a crucial effect on the solution. However, the l_1 regularization problem has no closed-form solution, and the regularization parameter is usually selected by experience. This study proposes two strategies of selecting the regularization parameter for the l_1-regularized damage detection problem. The first method utilizes the residual and solution norms of the optimization problem and ensures that they are both small. The other method is based on the discrepancy principle, which requires that the variance of the discrepancy between the calculated and measured responses is close to the variance of the measurement noise. The two methods are applied to a cantilever beam and a three-story frame. A range of the regularization parameter, rather than one single value, can be determined. When the regularization parameter in this range is selected, the damage can be accurately identified even for multiple damage scenarios. This range also indicates the sensitivity degree of the damage identification problem to the regularization parameter.

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1. Introduction

Vibration-based damage detection has received considerable attention in the past decades [1–3]. Finite element (FE) model updating can locate and quantify damage by utilizing structural vibration properties, such as natural frequencies and mode shapes [4–7].

The identification of structural damage based on measured modal parameters is essentially an inverse problem in mathematics and is typically ill-posed because of the large condition number of the sensitivity matrix [8]. Therefore, measurement noise would lead to inaccurate damage identification. In this regard, the regularization technique is employed by including a regularization term in the objective function, such that a physically meaningful and stable solution can be obtained [8]. Moreover, sensitivity-based model updating is underdetermined in the presence of infinite solutions because of

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the less number of identifiable modal parameters than that of structural elements. In this situation, the regularization technique is also required to obtain a unique solution.

Tikhonov regularization, also known as l_2 regularization, is a most commonly used scheme to stabilize the inverse problem and has been widely used in structural damage detection [9,10]. l_2 regularization has a closed-form solution and is thus efficient and convenient for implementation. However, this method tends to produce over-smooth solutions because the quadratic regularizer cannot recover the sharp features of the solution. Consequently, damage identification results are usually distributed to many structural elements, most of which are falsely identified as damage [11–13]. Similar results have also been reported in signal and visual reconstruction [14].

Civil structures generally contain a large number of elements or components whereas damage usually occurs at several sections or members only. Therefore, the damage index is zero for most elements, except for damaged ones. The damage index can be regarded as a sparse vector. According to sparse recovery theory, the l_1 regularization technique can effectively solve inverse problems, which possess sparsity in time or spatial domain [15]. This theory was initially employed in seismology and verified to perform better in preserving isolated characteristics than l_2 norm regularization [16]. Sparse recovery theory has gained substantial attention in the recent years for compressive sensing (CS) and has been widely applied in signal processing, wireless sensing, and image reconstruction [17,18]. This theory has also been applied to structural health monitoring (SHM) [19–22]. In particular, Zhou et al. [23] proposed a damage identification method by combining substructure-based sensitivity analysis and l_1 sparse regularization. Zhou et al. [11] developed a new damage detection method based on sparse recovery theory by using frequency data only. l_1 regularization was employed in FE model updating to improve the damage identifiability. The technique was later extended by including frequency and mode shapes [13]. Zhang et al. [24] presented a time-domain damage detection algorithm using the extended Kalman filter technique; l_1 regularization was imposed in the optimization process to suppress the interference of measurement noise and improve the identification accuracy.

In regularization methods, the regularization parameter plays a critical role by trading off the size of the regularized solution and how well it fits the given data [25]. A well-balanced regularization parameter can effectively deal with the ill-posedness of the inverse problem and yield a meaningful and stable solution. A number of methods have been developed to determine the optimal regularization parameter for inverse problems in mathematics; these methods include discrepancy principle (DP) [26–28], ordinary and generalized cross validations (GCV) [29], universal rules [30], and min–max rules [31]. The l_2 regularization has the closed-form solution; as such, tractable methods can be used to choose the regularization parameters [32], such as the widely used L-curve criterion [33]. However, the selection criterion of the regularization parameter for the l_1 regularization problem is very limited because the problem has no closed-form solution.

In SHM, an appropriate regularization parameter for the l_1-regularized problem is problem-dependent and typically selected by experience. Mascarenas et al. [34] set the regularization parameter as unit heuristically. Yang and Nagarajaiah [35] reported the insensitivity of the solution to the regularization parameter and set it as 0.01 in CS-based modal identification. Another study [36] calculated the regularization parameter using $\beta = 1/\sqrt{N}$ (where N is the number of the time history sampling points corresponding to the dimension of the unknown vector). Zhang and Xu [12] chose the regularization parameter by using the re-weighted l_1 regularization technique. Yao et al. [37] showed that the plot of the residual term versus the regularization term on the linear scale resembled an “L” shape; afterward, they selected the regularization parameter corresponding to the corner of the L curve.

In this study, two strategies are developed for selecting the regularization parameter for the l_1-regularized problem. Inspired by the L-curve criterion used in the l_2 regularization, the first strategy utilizes the residual and solution norms of the optimization problem to determine an appropriate range of the regularization parameter for the problem. The second strategy is based on the DP used in the l_2 counterpart. The two techniques yield consistent results. Their effectiveness is demonstrated through applications to a laboratory tested cantilever beam and a steel frame.

2. Sensitivity-based damage detection using l_1 regularization

In structural damage identification, the global stiffness matrix of an undamaged structure can be expressed as follows [2,11].

$$[K] = \sum_{i=1}^{n} \alpha_i \left[K^i\right] \tag{1}$$

where $[K^i]$ is the ith element stiffness matrix, α_i is the element stiffness parameter, and n is the number of elements. Under the assumption that only the element stiffness is reduced when damage occurs, the structural stiffness matrix in the damaged state takes the following form

$$[K] = \sum_{i=1}^{n} (1 + p_i) \left[K^i\right] \tag{2}$$

where p_i is the stiffness reduction factor (SRF) and defined as [11,38].

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