Journal of Sound and Vibration 423 (2018) 401–420

Contents lists available at ScienceDirect

Journal of Sound and Vibration

journal homepage: www.elsevier.com/locate/jsvi

Dynamic identification of axial force and boundary restraints in tie rods and cables with uncertainty quantification using Set Inversion Via Interval Analysis

Timothy Kernicky [a], Matthew Whelan [a,*], Ehab Al-Shaer [b]

[a] University of North Carolina at Charlotte, Department of Civil and Environmental Engineering, 9201 University City Boulevard, Charlotte, NC 28223-0001, USA
[b] University of North Carolina at Charlotte, Department of Software and Information Systems, 9201 University City Boulevard, Charlotte, NC 28223-0001, USA

ARTICLE INFO

Article history:
Received 6 September 2017
Revised 9 February 2018
Accepted 27 February 2018
Available online XXX

Keywords:
Cable force
Parameter identification
Inverse methods
Uncertainty quantification
Vibration testing
Boundary conditions

ABSTRACT

A methodology is developed for the estimation of internal axial force and boundary restraints within in-service, prismatic axial force members of structural systems using interval arithmetic and contractor programming. The determination of the internal axial force and end restraints in tie rods and cables using vibration-based methods has been a long standing problem in the area of structural health monitoring and performance assessment. However, for structural members with low slenderness where the dynamics are significantly affected by the boundary conditions, few existing approaches allow for simultaneous identification of internal axial force and end restraints and none permit for quantifying the uncertainties in the parameter estimates due to measurement uncertainties. This paper proposes a new technique for approaching this challenging inverse problem that leverages the Set Inversion Via Interval Analysis algorithm to solve for the unknown axial forces and end restraints using natural frequency measurements. The framework developed offers the ability to completely enclose the feasible solutions to the parameter identification problem, given specified measurement uncertainties for the natural frequencies. This ability to propagate measurement uncertainty into the parameter space is critical towards quantifying the confidence in the individual parameter estimates to inform decision-making within structural health diagnosis and prognostication applications. The methodology is first verified with simulated data for a case with unknown rotational end restraints and then extended to a case with unknown translational and rotational end restraints. A laboratory experiment is then presented to demonstrate the application of the methodology to an axially loaded rod with progressively increased end restraint at one end.

1. Introduction

Short cables and hangers in tied arch bridges, struts of truss structures, diagonal bracing in building frames, and tie rods are examples of axial force members in structural systems where knowledge of the actual in-service levels of axial force can significantly aid in performance evaluation and health monitoring of the structure. Estimation of cable tension in bridge suspenders and hangers using measured natural frequencies has been long established as a nondestructive evaluation technique [1–3],

* Corresponding author.
E-mail addresses: tkernick@uncc.edu (T. Kernicky), M.Whelan@uncc.edu (M. Whelan), ealshaer@uncc.edu (E. Al-Shaer).

https://doi.org/10.1016/j.jsvi.2018.02.062

Nomenclature

CSP	constraint satisfaction problem
SIVIA	set inversion via interval analysis
Intervals	
$[x]$	scalar interval
$[\boldsymbol{x}]$	interval vector (or box)
$\underline{x}$	lower bound of $[x]$
$\overline{x}$	upper bound of $[x]$
$w([x])$	width of $[x]$
ϵ	prescribed interval precision
Sets	
$\emptyset$	empty set
$\mathbb{S}$	generic set
$\underline{\mathbb{S}}$	inner approximation of $\mathbb{S}$
$\overline{\mathbb{S}}$	outer approximation of $\mathbb{S}$
$\delta\mathbb{S}$	boundary of $\mathbb{S}$
$\mathbb{R}$	set of all real numbers
$\mathbb{IR}$	set of all interval real numbers
Symbols	
$\cap$	intersection
$\cup$	union
$\sqcup$	interval union
$\subset$	subset
χ	measurement uncertainty
N	axial force

while the use of dynamic cable force identification as a tool for structural health monitoring of the onset of structural damage in bridges, by evaluating the redistribution of axial forces in bridge cables, has also been investigated [4]. However, estimation of tension forces in cables and other axial force members with low slenderness, where the bending stiffness of the cable and boundary condition degree of fixity significantly affect the progression of natural frequencies, yields a challenging nonlinear and potentially ill-conditioned inverse problem that remains an active area of research with many practical applications.

In addition to axial force estimation, determination of the end restraint conditions in structural members provides numerous advantages within performance assessment and structural health monitoring. Axial force members are often designed to operate with unrestrained or nominally restrained end rotations. When environmental effects, such as corrosion, introduce unintended end restraint, the structural member may be subject to significant flexural stresses that may lead to fatigue cracking and subsequent failure. There are numerous examples of infrastructure failures driven by seizure of pins, including successive failure of several tied arch hangers in the Sgt. Aubrey Cosens VC Memorial Bridge [5] and the collapse of the Mianus River Bridge following failure of pin and hanger assemblies [6]. Reliable in-service methods for determining the effective end restraints on axial force members would provide a means for nondestructively testing critical components over the service life to inform corrective actions for mitigating long-term performance issues.

However, while numerous approximation formulas and iterative techniques [2] exist for the determination of axial forces in long slender cables using dynamic measurements, only recently have techniques emerged for the estimation of axial forces in members with significant bending stiffness. Many cable force estimation techniques rely on approximations to the solution of the characteristic equation that are not valid for short cables and beams, where the bending stiffness introduces significant nonlinearity in the progression of natural frequencies [1]. Furthermore, the dynamic response of members with high relative bending stiffness is significantly affected by the restraint provided by end connections, unlike slender cables where the relative approximation error resulting from idealization of boundary conditions is often negligible. In these cases, the experimental parameter identification must include the unknown axial force and the boundary condition restraints at the ends of the member. The need to identify these additional coupled parameters make this nonlinear inverse problem significantly more challenging and, to date, only a limited number of techniques based on the use of measured natural frequencies have been proposed for members with significant bending stiffness. Ceballos and Prato [7] developed a technique for determining the axial force and bending stiffness using least squares minimization. However, this method is limited to cables with identical fixity on each end and, furthermore, the end restraints are not solved for simultaneously with the internal axial force, which may lead to significant errors due to the parameter dependency in this potentially ill-conditioned problem. Within tie-rod applications, analytical models have generally adopted a simply supported beam representation with either elastic rotational end springs or elastic Winkler foundation springs over some length of the beam ends to account for the support condition provided by anchorage. Parameter identification of the axial force and stiffness of the elastic springs has been proposed through model updating [8].

Recently, a technique has been developed to determine the internal axial force and boundary conditions of slender beams using a minimum of five displacement or curvature measurements obtained along the length of the beam under the action of known static loading [9]. The technique was extended to dynamic measurements by leveraging a single flexural mode shape estimate obtained with no less than five sensors and the corresponding natural frequency measurement [10]. Li et al. [11] presented a similar approach that uses singular value decomposition to solve for unknowns in the characteristic equation to develop estimates of axial force and boundary restraints using a distributed array of five accelerometers, strain gages, or displacement transducers. Timoshenko beam theory has been recently introduced to these five transducer approaches to account for the effects of shear deformation and rotary inertia [12]. These approaches, however, require a fairly large number of sensors