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Vibration due to non-circularity of a rotating ring having discrete radial supports - With application to thin-walled rotor/magnetic bearing systems

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ABSTRACT

This paper investigates the vibration arising in a thin-walled cylindrical rotor subject to small non-circularity and coupled to discrete space-fixed radial bearing supports. A Fourier series description of rotor non-circularity is incorporated within a mathematical model for vibration of a rotating annulus. This model predicts the multi-harmonic excitation of the rotor wall due to bearing interactions. For each non-circularity harmonic there is a set of distinct critical speeds at which resonance can potentially arise due to flexural mode excitation within the rotor wall. It is shown that whether each potential resonance occurs depends on the multiplicity and symmetry of the bearing supports. Also, a sufficient number of evenly spaced identical supports will eliminate low order resonances. The considered problem is pertinent to the design and operation of thin-walled rotors with active magnetic bearing (AMB) supports, for which small clearances exist between the rotor and bearing and so vibration excitation must be limited to avoid contacts. With this motivation, the mathematical model is further developed for the case of a distributed array of electromagnetic actuators controlled by feedback of measured rotor wall displacements. A case study involving an experimental system with short cylindrical rotor and a single radial AMB support is presented. The results show that flexural mode resonance is largely avoided for the considered design topology. Moreover, numerical predictions based on measured non-circularity show good agreement with measurements of rotor wall vibration, thereby confirming the validity and utility of the theoretical model.

1. Introduction

The vibration behaviour of circular shell structures has been studied extensively over the last few decades as it is important for many conventional and emerging mechanical systems. In particular, current architectures in rotating machine design are evolving towards lightweight hollow rotor structures, including aero engines [1,2]. The vibration within the walls of a simple thin-walled rotor shares similarities with the in-plane vibrations of rings, for which Coriolis and centripetal accelerations give rise to rotational-speed-dependent splitting of natural frequencies for forward and backward circumferential waves [3,4]. The motivation for the present study was to better understand the underlying mechanism for vibrational excitation of a thin-walled rotor supported by active magnetic bearings. Vibration is seen to arise due to imperfect symmetry of the rotor cross-section and its accurate prediction may play an important role in machine design, manufacture and operation. A suitable dynamic model

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must incorporate mathematical descriptions of rotor non-circularity and space-fixed bearing elements in order to establish the forced vibration behaviour under rotation.

Limitations in manufacturing processes will cause profile variations in hollow cylindrical structures of nominally annular cross-section. Although a number of studies on the free vibration of imperfect rings can be found in the literature, these deal almost exclusively with non-rotating rings. Imperfections may be introduced as small initial displacements [5], or variation of the ring cross section [6,7], or perturbations of the uniform mass density and bending stiffness [8]. An elliptical ring has also been considered as a special case of an imperfect circular ring [9]. In most of these studies, the Rayleigh-Ritz approach has been used to determine the perturbations in natural frequencies and mode shapes for free vibration. The effect of uneven mass distribution for a spinning resonator ring in a MEMs device was studied in Ref. [10].

Machine rotor/shaft structures are usually supported by space-fixed bearing components with certain stiffness and damping characteristics. For a thin-walled rotor, bearing interactions occurring at discrete angular positions may result in significant changes in modal properties for wall vibration. The related problem of free vibration of a non-rotating thin ring on a general elastic foundations was solved in Ref. [11] via both perturbation and Galerkin methods. Similar behaviour is seen in models of vibration in meshing compliant gears which couple through space-fixed discrete stiffnesses [12,13].

The vibration of a rotating ring under forced conditions, both with and without constraints, was studied by Carrier [14] for a few special cases. Closed form solutions to the harmonic and periodic forced vibration of rotating rings have been reported by Huang and Soedel [15]. Response solutions for rotating tires based on rings with elastic foundations under various loading situations have also been published [16,17], while an experimental study on tire vibration can be found in Ref. [18].

Previous studies on forced vibration have been limited to the case of perfect rings subject to exogenous forcing. To the authors' knowledge, the mechanism for self-excitation due to imperfect geometry of a hollow cylindrical rotor with discrete supports has not previously been dealt with. In section 2 of this paper, a model is developed for vibration of a short thin-walled rotor where non-circularity is represented by an initial deflection of the rotor wall. Using a Fourier series description of non-circularity, a general model for the interaction of the rotor with space-fixed discrete supports is derived. Two types of bearing supports are considered:

1. An array of radial spring-damper bearing elements
2. An active bearing involving an array of compact electromagnetic actuators

In section 3, a numerical study is undertaken to investigate the influence of multiplicity and symmetry of discrete spring-damper supports on the resonance behaviour of the rotor. In section 4, an experimental investigation is described involving a prototype active magnetic bearing (AMB) with distributed actuation applied to a thin-walled rotor. Measurements of vibration excitation during operation are presented and compared with results from theoretical modelling. The final section draws conclusions.

2. Mathematical modelling

2.1. Equation of motion

The geometry of a thin-walled rotor with small non-circularity is shown in Fig. 1. The rotor has circumference length L_c and rectangular cross-section of depth d and length l. Uniform density ρ and modulus of elasticity E are assumed. The rotor is supported by space-fixed discrete bearing elements, shown here as radial spring-damper units. The XY axes are a fixed reference frame, while the reference frame X′Y′ rotates with the rotor at constant angular speed Ω about the fixed axis through O.

To derive the equations of motion we consider that non-circularity is introduced as an initial plastic deformation of a perfect circular ring so that its neutral plane is perturbed from a reference circle centred on the coordinate axes, as shown in Fig. 1a. Then, additional displacements arise under motion due to elastic deformation. The local reference frame for the displacement at a given point P is shown by axes x_0y_0 and the deformation frame is denoted by axes xy. The displacements of the cylinder wall in radial (x_0) direction and tangential (y_0) direction with respect to the reference circle are defined as

$$U(\theta,t) = u_{r0}(\theta) + u_r(\theta,t), \quad V(\theta,t) = v_{r0}(\theta) + v_r(\theta,t)$$

where $u_{r0}(\theta)$ and $v_{r0}(\theta)$ are the initial equilibrium displacements due to non-circularity and $u_r(\theta,t)$ and $v_r(\theta,t)$ are the displacements due to vibration. Under these assumptions we may apply an inextensibility condition to both the plastic and elastic components of the deformation:

$$u_r = v_r', \quad u_{r0} = v_{r0}' \tag{1}$$

Assuming that the cross-section planes remain plane after deformation, the angle between deformed and undeformed cross-section is $\varphi = \frac{1}{R}(U' + V)$, where the prime designates the partial derivative with respect to angle θ. The position vector for point P can be written as $\mathbf{r}_P = -(R-U)\mathbf{i}_0 + V\mathbf{j}_0$ and the acceleration as $\ddot{\mathbf{r}}_P = \ddot{r}_{0x}\mathbf{i}_0 + \ddot{r}_{0y}\mathbf{j}_0$ where $\ddot{r}_{0x} = \ddot{U} + 2\Omega\dot{V} + \Omega^2(R-U)$ and $\ddot{r}_{0y} = \ddot{V} - 2\Omega\dot{U} - \Omega^2 V$. Neglecting nonlinear terms, the acceleration with respect to the xy-frame is $\ddot{\mathbf{r}}_P = \ddot{r}_x\mathbf{i} + \ddot{r}_y\mathbf{j}$ where $\ddot{r}_x = \ddot{U} + 2\Omega\dot{V} + \Omega^2(R-U)$ and $\ddot{r}_y = \ddot{V} - 2\Omega\dot{U} + \Omega^2 U'$. Moreover, the inertia forces are $f_x = \rho dlR\ddot{r}_x d\theta$ and $f_y = \rho dlR\ddot{r}_y d\theta$. Hence, the equations of motion for a differential element with length $ds = Rd\theta$ may be obtained by resolving in the instantaneous